Dyadic T-mesh Subdivision

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Dyadic T-mesh Subdivision
T-mesh
Why T-meshes?
Varying Amount of Detail
Spinning Top
Spinning Top

- Edge alignment with curvature directions
- Either too few control points on outside
- Or too many control points on inside
Spinning Top

- Edge alignment with curvature directions
- Either too few control points on outside
- Or too many control points on inside
Spinning Top

- Needs T-vertex-type topology
  - Not in Catmull-Clark vocabulary, treated as poles (= extraordinary vertices)
  - Edges not aligned with curvature directions
  - Impacts surface quality
• Needs T-vertex-type topology
• Not in Catmull-Clark vocabulary, treated as poles (= extraordinary vertices)
• Edges not aligned with curvature directions
• Impacts surface quality
Spinning Top

- Needs T-vertex-type topology
- Not in Catmull-Clark vocabulary, treated as poles
  (= extraordinary vertices)
  - Edges not aligned with curvature directions
  - Impacts surface quality
Spinning Top

- Needs T-vertex-type topology
- Not in Catmull-Clark vocabulary, treated as poles (= extraordinary vertices)
- Edges not aligned with curvature directions
- Impacts surface quality
Spinning Top

- Needs T-vertex-type topology
- Not in Catmull-Clark vocabulary, treated as poles (= extraordinary vertices)
- Edges not aligned with curvature directions
- Impacts surface quality
Redirected Edge Flow
Parametric domain

Knot Intervals
Parametric domain
Parametric domain
Parametric domain
Parametric domain
uniform knot intervals
Non-uniform knot intervals
Non-uniform knot intervals

Remember This!
Dyadic T-mesh Subdivision
Spinning Top: T-mesh Subdivision

- Need for T-vertex-type topology
- And T-mesh subdivision scheme
Spinning Top: T-mesh Subdivision

- Need for T-vertex-type topology
- And T-mesh subdivision scheme
Spinning Top: T-mesh Subdivision

- Need for T-vertex-type topology
- And T-mesh subdivision scheme
• **Recursively generated B-spline surfaces on arbitrary topological meshes** (1978).
  Catmull, E., and Clark, J.

• **Interactive multiresolution mesh editing** (1997).
  Zorin, D., Schröder, P., and Sweldens, W.

• **Non-uniform recursive subdivision surfaces** (1998).
  Sederberg, T., Zheng, J., Sewell, D., and Sabin, M.

• **T-splines and T-NURCCs** (2003).

• **Polynomial splines over general T-meshes** (2010).
  Li, X., Deng, J., and Chen, F.

• **On linear independence of T-spline blending functions** (2012).
  Li, X., Zheng, J., Sederberg, T., Hughes, T., and Scott, M.
• **Recursively generated B-spline surfaces on arbitrary topological meshes** (1978).
  Catmull, E., and Clark, J.

• **Interactive multiresolution mesh editing** (1997).
  Zorin, D., Schröder, P., and Sweldens, W.

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Tensor-Product Surfaces

Spline Surfaces

Uniform B-Splines

Subdivision Surfaces

Catmull-Clark Subdivision

Non-Tensor-Product Surfaces
B-Spline Surfaces: Knot Doubling
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B-Spline Surfaces: Knot Doubling
Catmull-Clark Factorization
Catmull-Clark Factorization
Catmull-Clark Factorization
Catmull-Clark Factorization

\[ \frac{1}{4} \]

\[ F_1 \]

\[ \frac{1}{2} \]

\[ M \]

\[ \frac{1}{4} \]

\[ F_2 \]
Catmull-Clark Factorization
Catmull-Clark Factorization generalizes to arbitrary topologies.
Tensor-Product Surfaces

Spline Surfaces

Uniform B-Splines

Knot Doubling / Factorization

Subdivision Surfaces

Catmull-Clark Subdivision

Non-Tensor-Product Surfaces
Non-uniform knot intervals

Remember This!
NURSS

- Non-uniform recursive subdivision surfaces
- No T-vertices
- Generalization of Catmull-Clark with non-uniform knot intervals
NURSS Factorization

\[ E \]

\[ w_1 \quad w_2 \quad w_3 \quad w_4 \]

\[ F_1 \quad M \quad F_2 \]

\[ E \]
Uniform B-Splines → Non-uniform B-Splines

Tensor-Product Surfaces

Spline Surfaces

Uniform B-Splines

Knot Doubling / Factorization

Non-uniform B-Splines

Subdivision Surfaces

Catmull-Clark Subdivision → NURSS
Tensor-Product Surfaces

- Uniform B-Splines
- Knot Doubling / Factorization
  - Catmull-Clark Subdivision
  - NURSS

Non-Tensor-Product Surfaces

- Non-uniform B-Splines
- AST-Splines
  - Dyadic T-mesh Subdivision

Spline Surfaces

Subdivision Surfaces
For Which T-meshes Is Knot Doubling Possible?

Analysis-Suitable
For Which T-meshes Is Knot Doubling Possible?

Analysis-Suitable
For Which T-meshes Is Knot Doubling Possible?

Dyadic
For Which T-meshes Is Knot Doubling Possible?

Dyadic
Dyadic AST-Spline Knot Doubling
Dyadic AST-Spline Knot Doubling
Dyadic AST-Spline Knot Doubling
Generalizing Factorization to T-meshes
Generalizing Factorization to T-meshes
Generalizing Factorization to T-meshes
Generalizing Factorization to T-meshes

1. Regularizing stencils

2. T-joint weight adjustments
1) Regularizing stencils
1) Regularizing stencils
1) Regularizing stencils
1) Regularizing stencils
1) Regularizing stencils
1) Regularizing stencils
Close, but not quite
2) T-joint weight adjustments

- T-joints in the stencil result in weights shifted to different vertices
2) T-joint weight adjustments

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- T-joints in the stencil result in weights shifted to different vertices

$$w_1 \quad w_2$$

$$w_3 \quad w_4$$
2) T-joint weight adjustments

- T-joints in the stencil result in weights shifted to different vertices

\[ w_2 \]

\[ w_1 + w_4 \]

\[ w_3 \]
Extraordinary Vertices

• Propose fix NURSS rules (sometimes loss of tangent plane continuity)

• Experimentally, C1 for many configurations and knot weights
Examples

• Maya-Plugin and MATLAB scripts at http://denkovacs.com/publications/dyadic-t-mesh-subdivision/
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