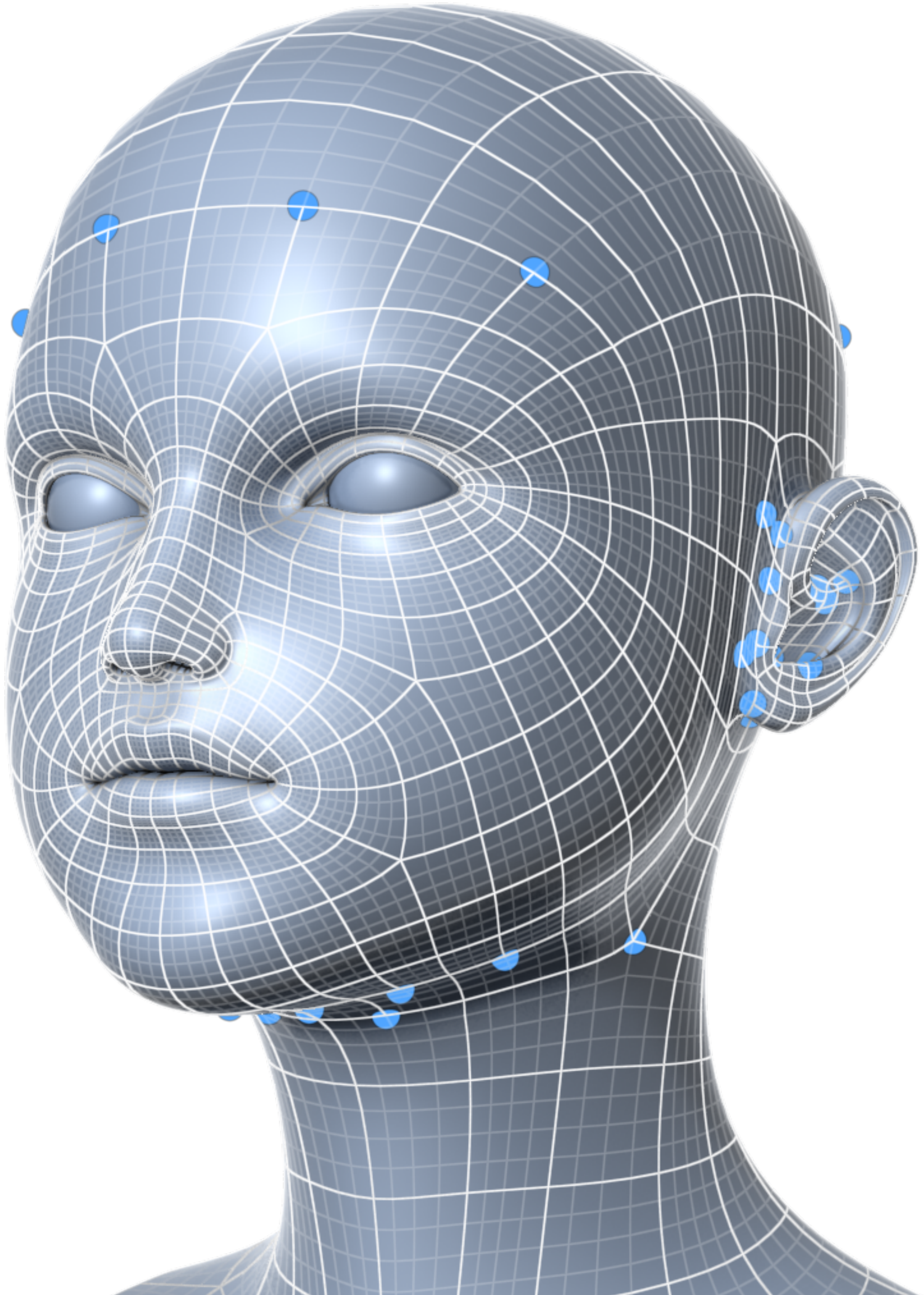
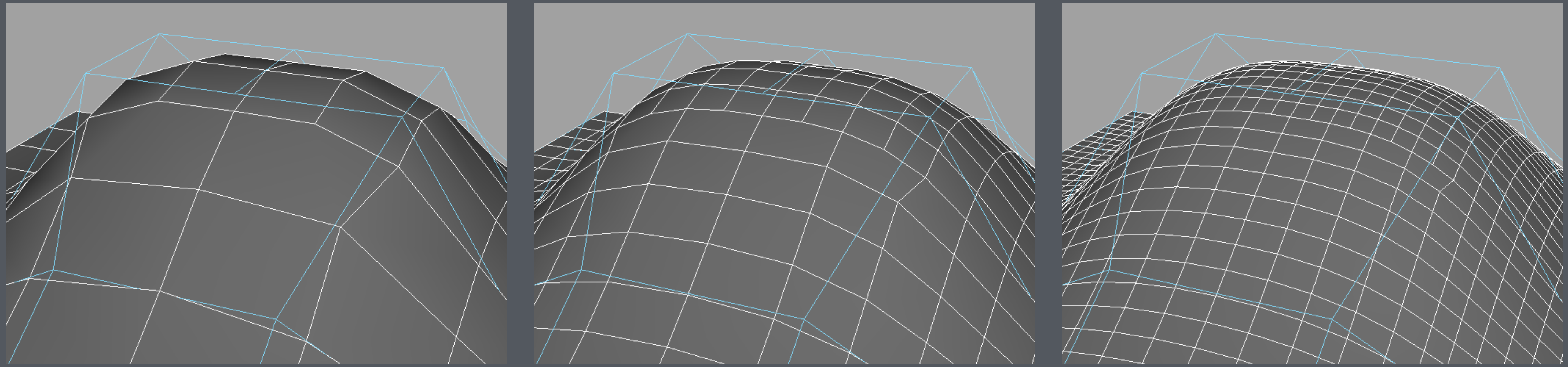
# SIGGRAPH 2015

Xroads of Discovery





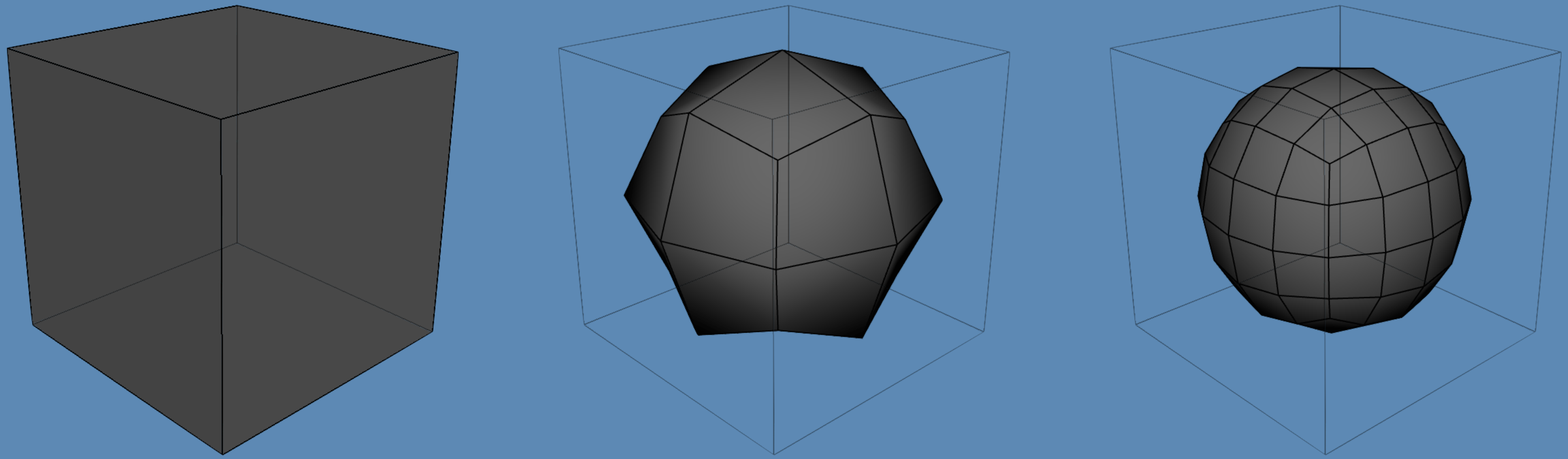
# Dyadic T-mesh Subdivision

Denis Kovacs - *New York University / FiftyThree*

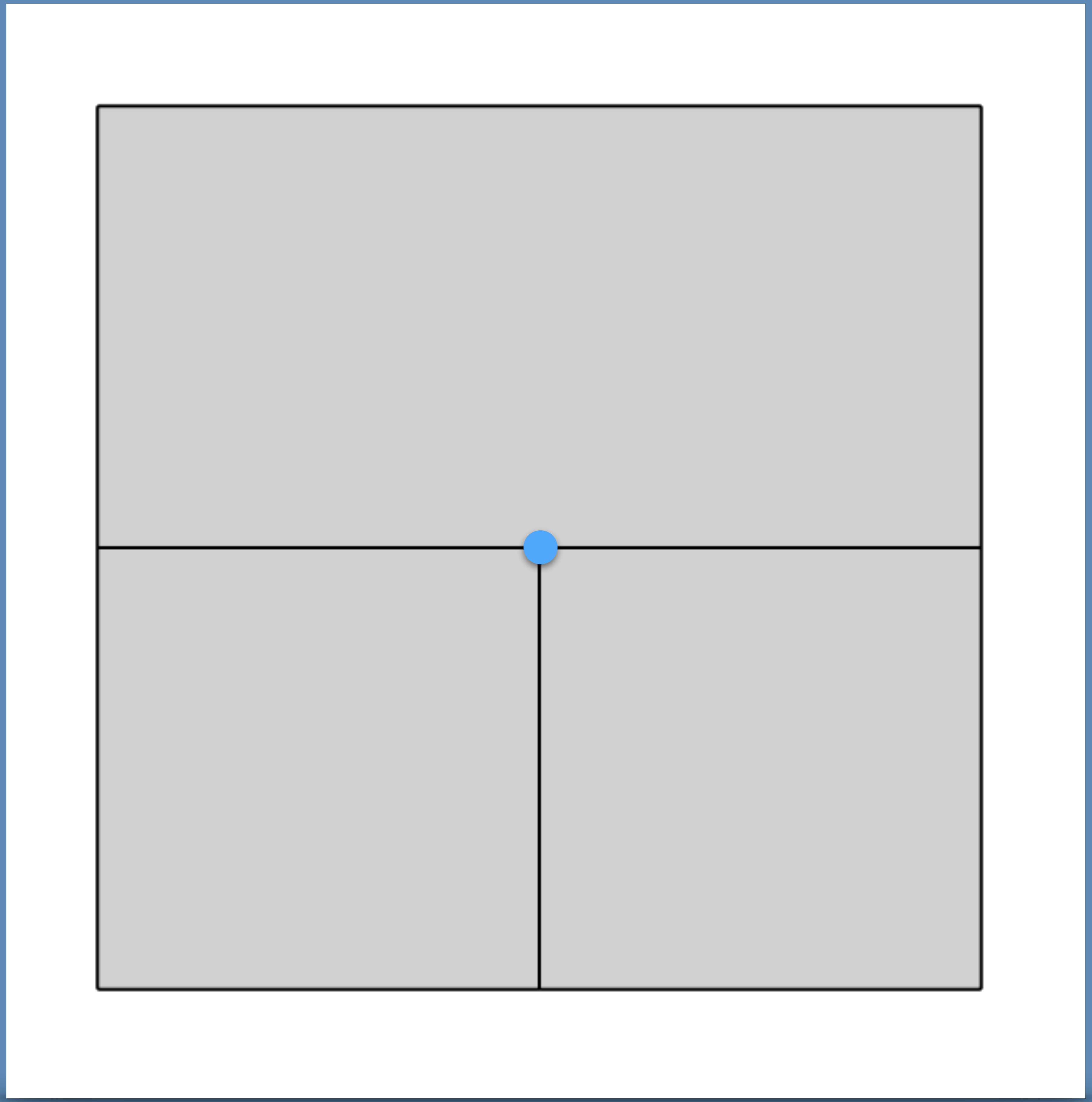
Justin Bisceglio - *New York University / Blue Sky Studios*

Denis Zorin - New York University

# Dyadic T-mesh Subdivision

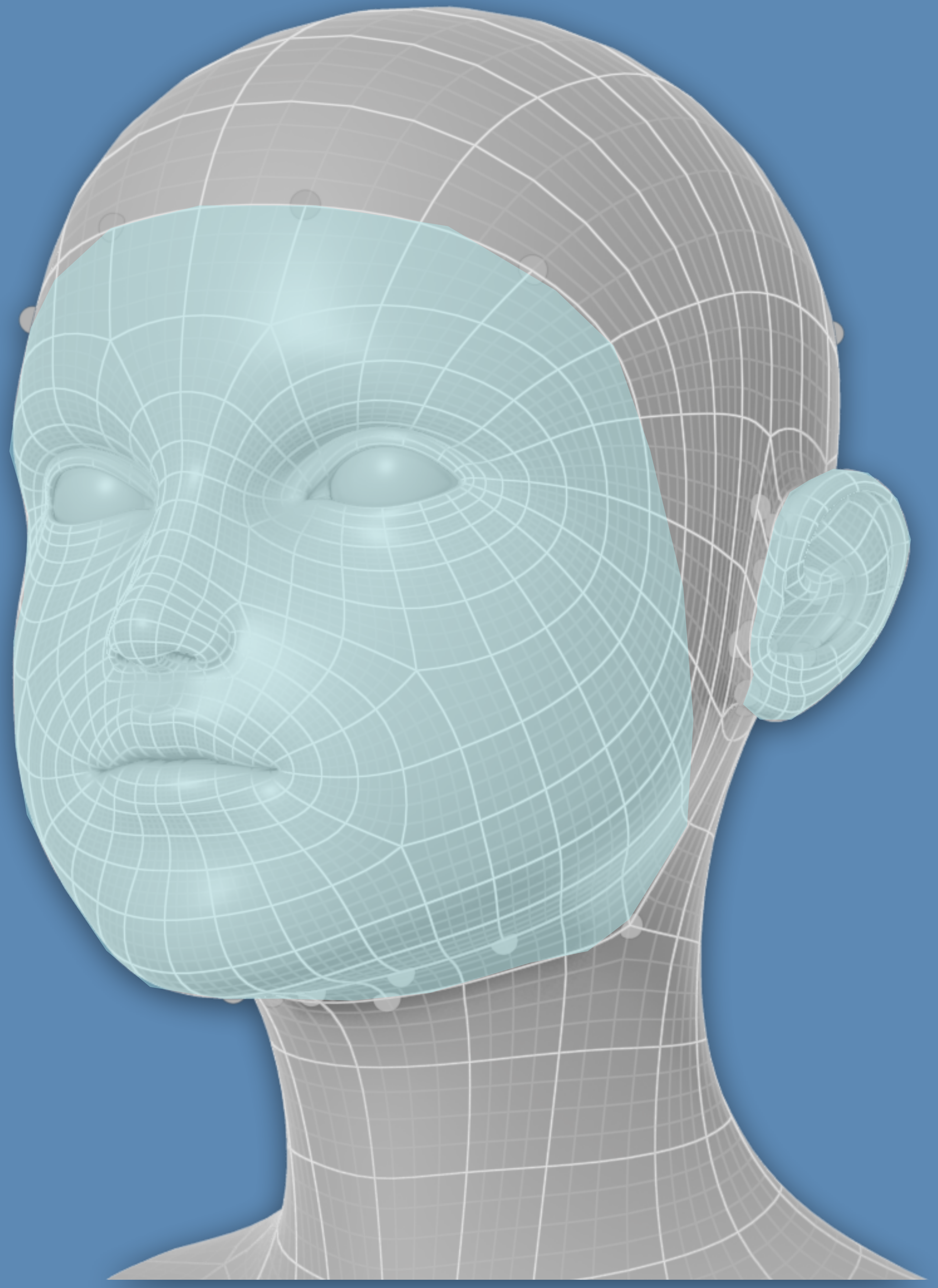


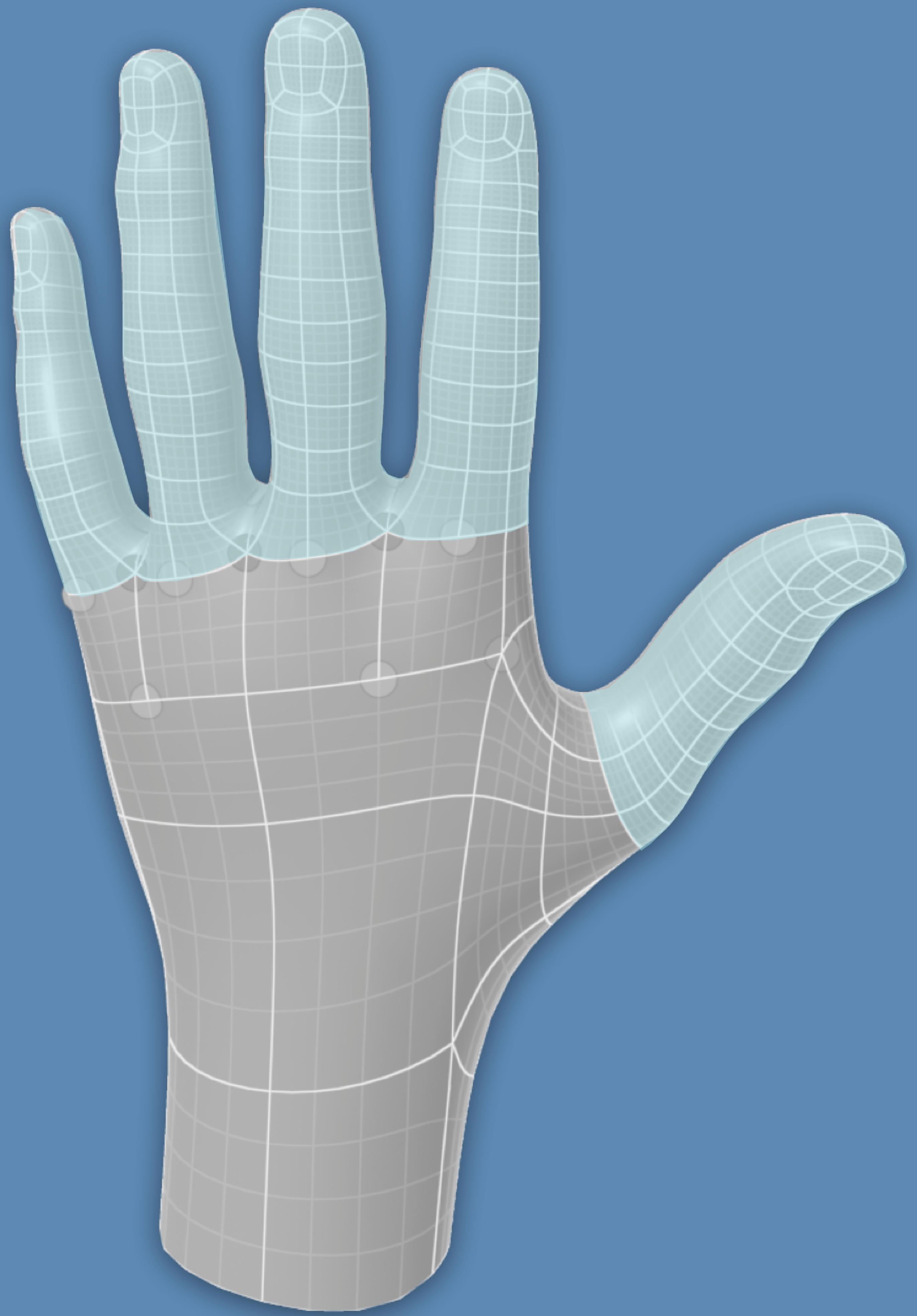
# T-mesh



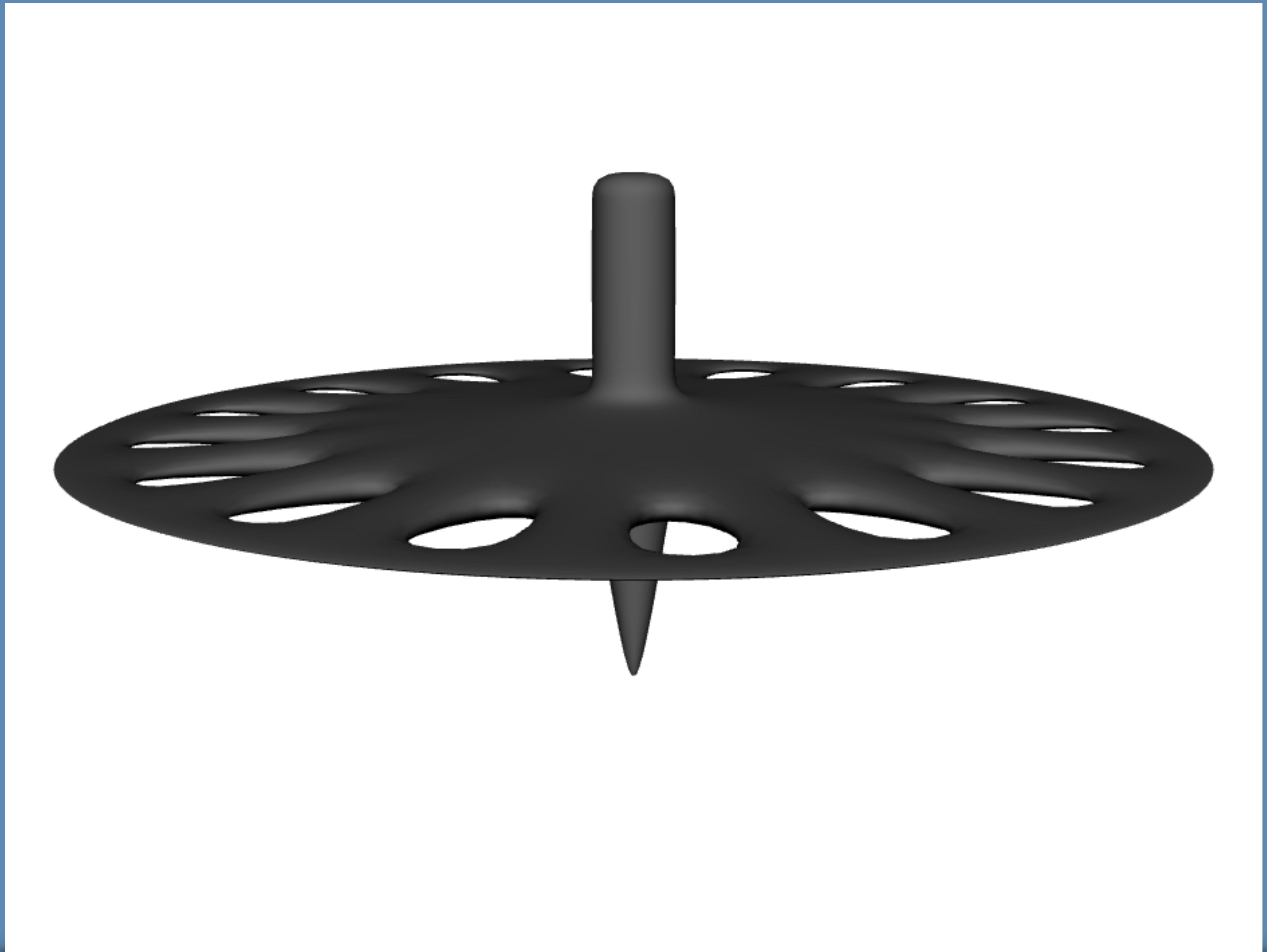
# Why T-meshes?

# Varying Amount of Detail





# Spinning Top



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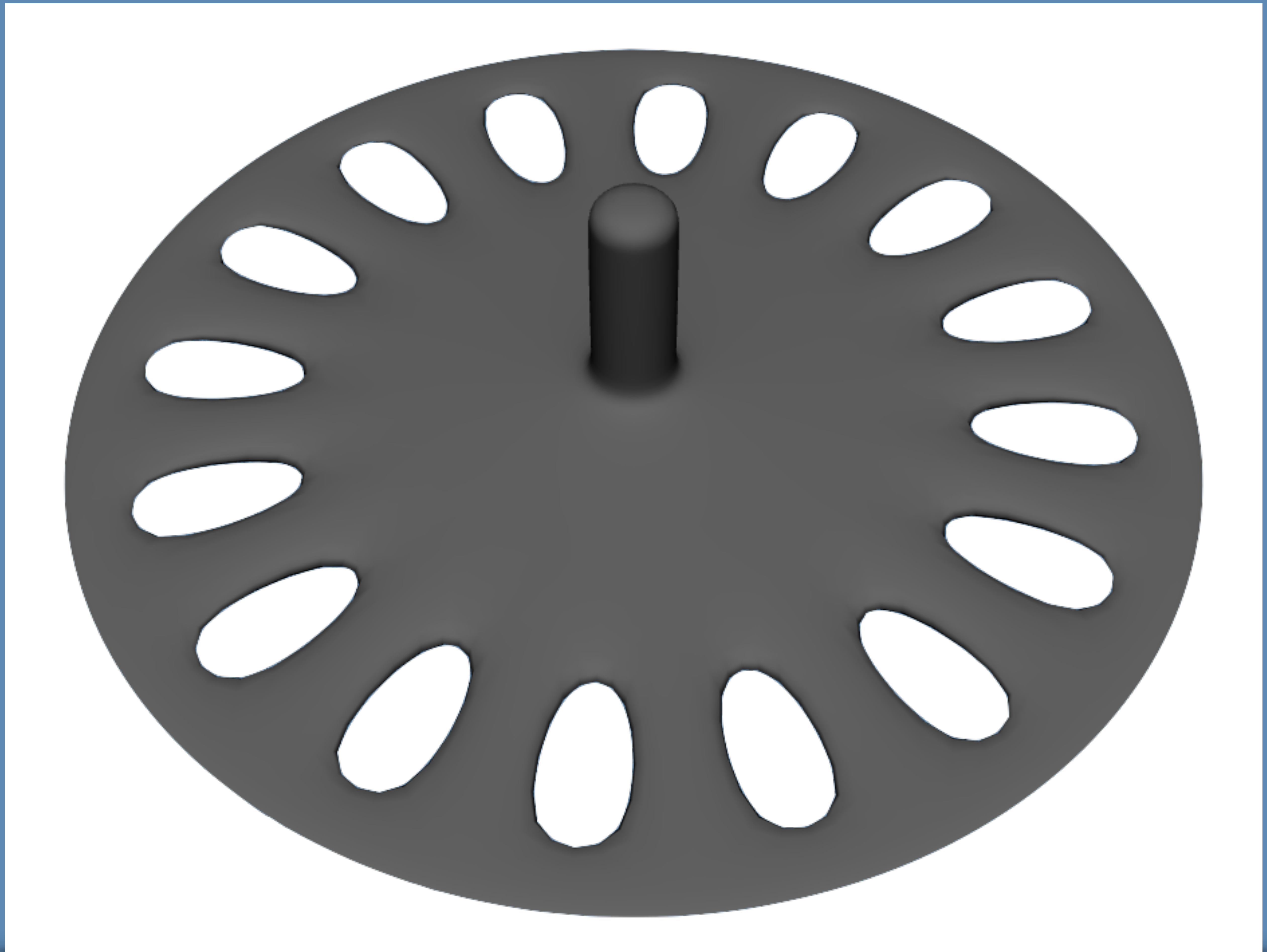
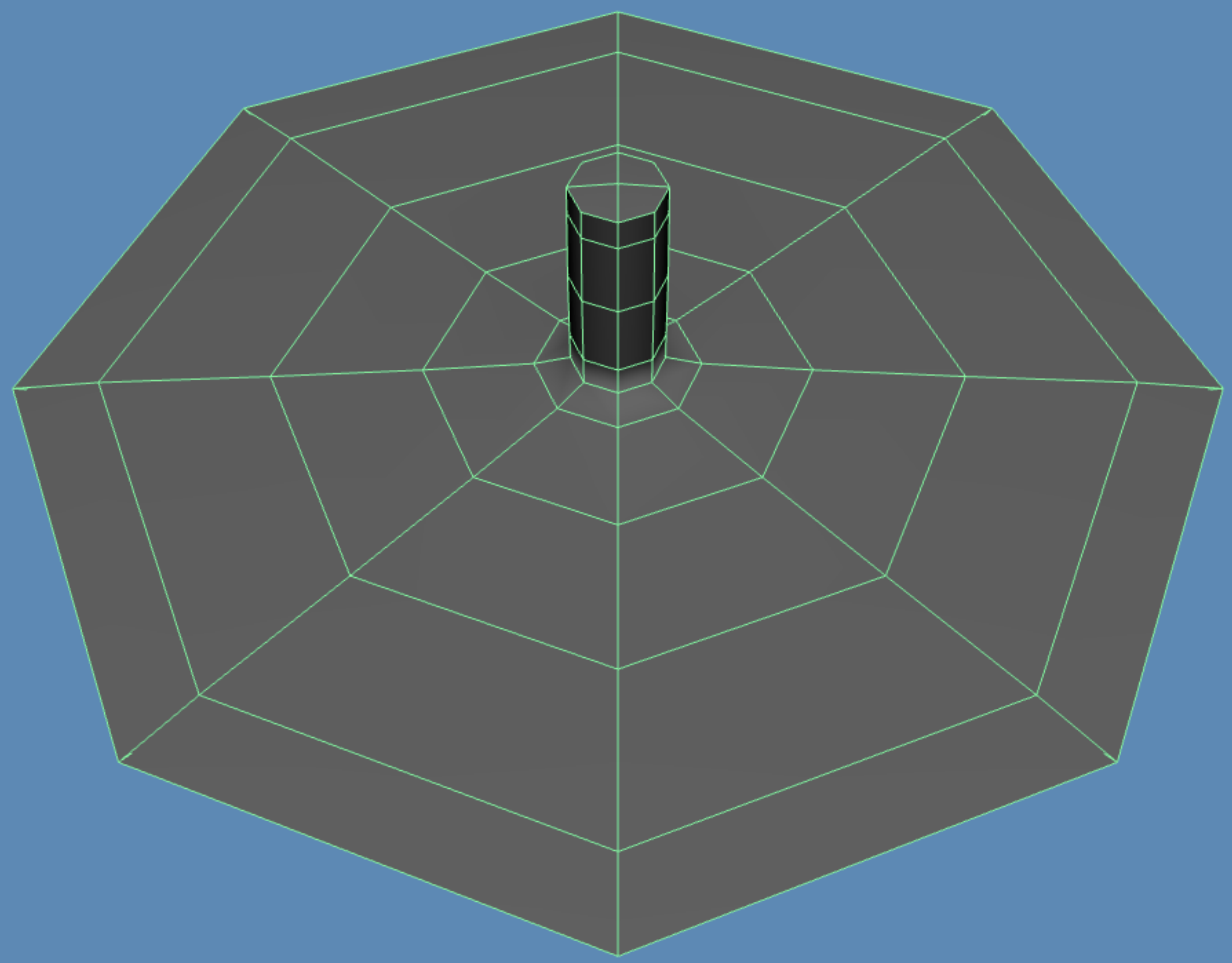


# Spinning Top

- Edge alignment with curvature directions
- Either too few control points on outside
- Or too many control points on inside



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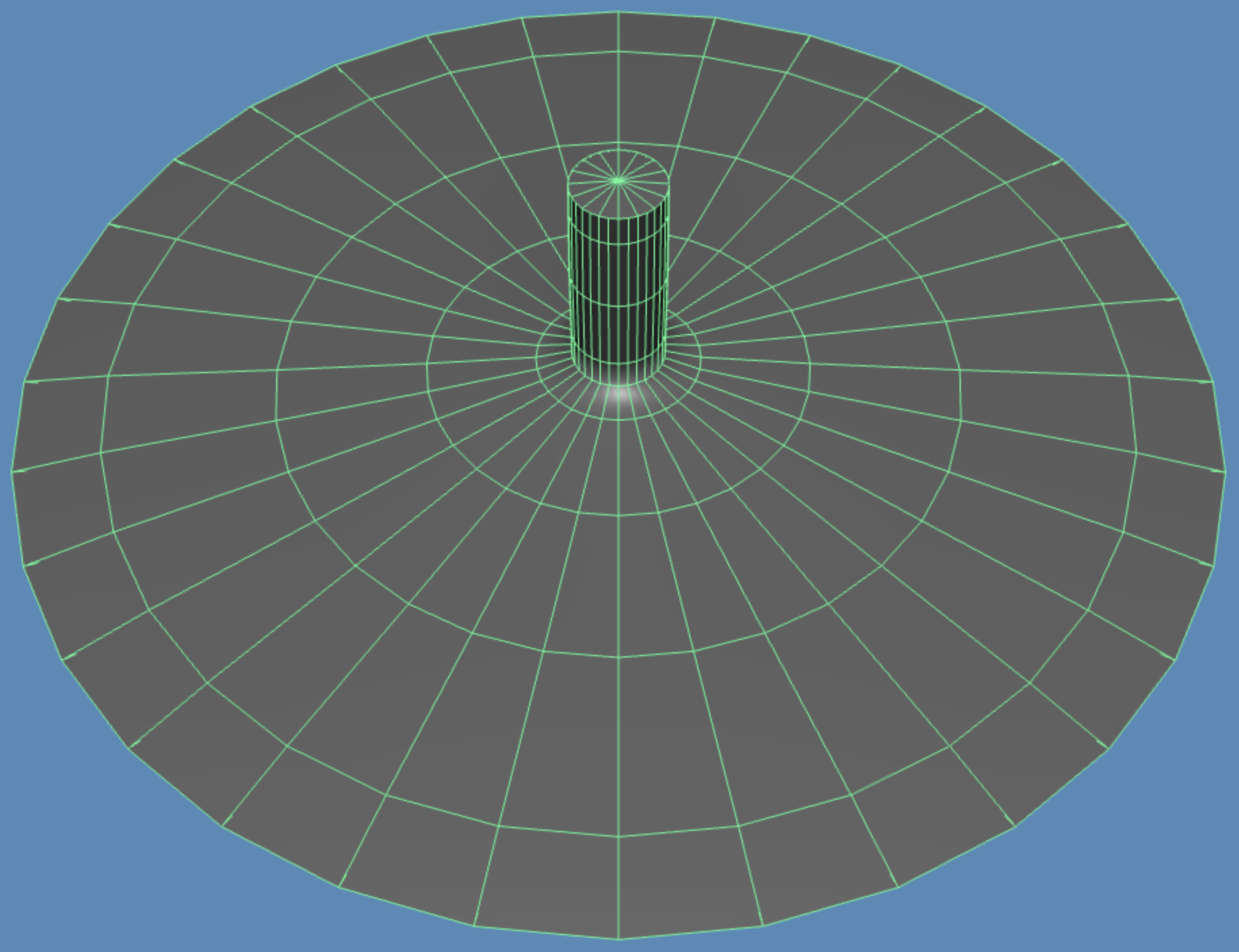
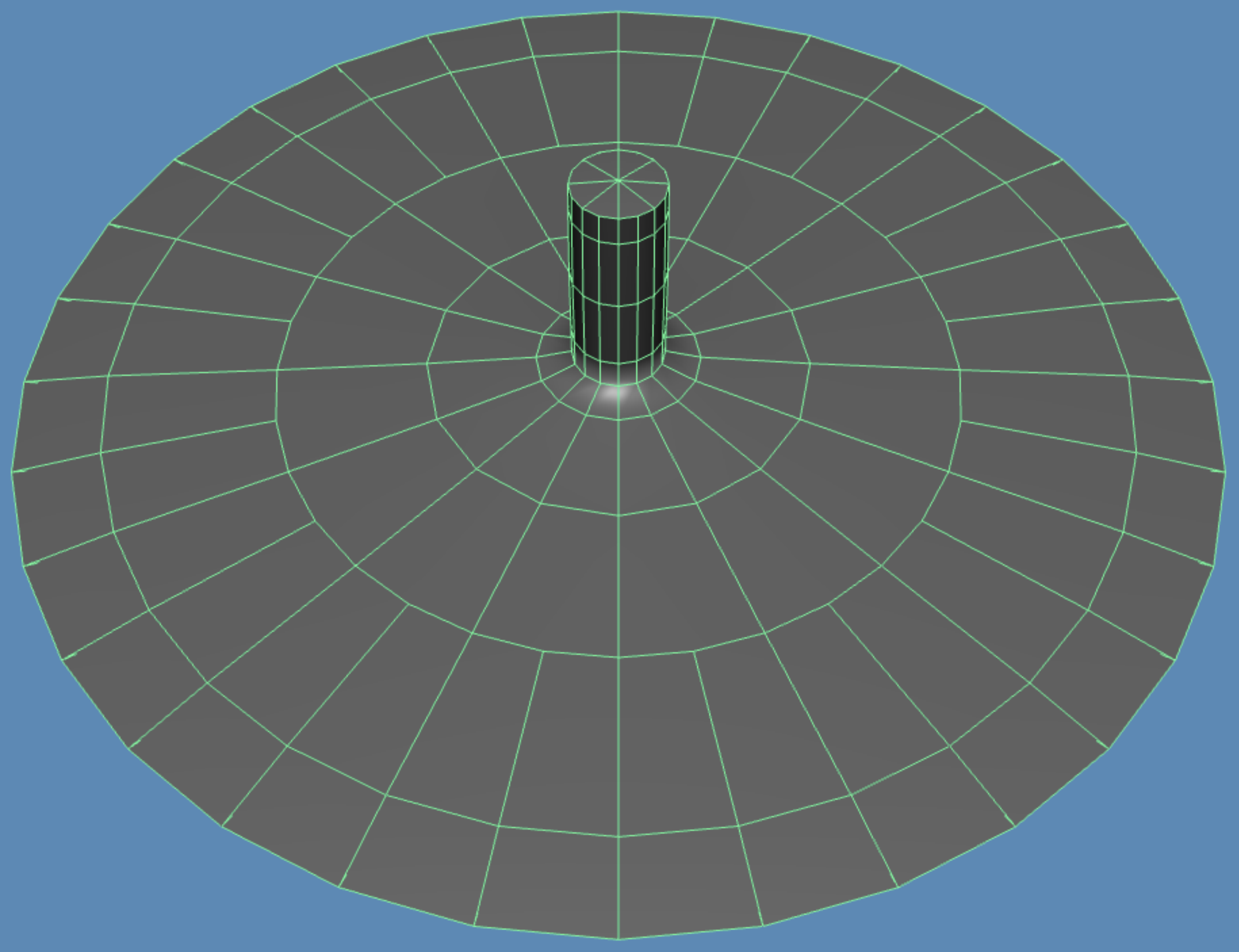


# Spinning Top

- Needs T-vertex-type topology
- Not in Catmull-Clark vocabulary, treated as poles (= extraordinary vertices)
- Edges not aligned with curvature directions
- Impacts surface quality



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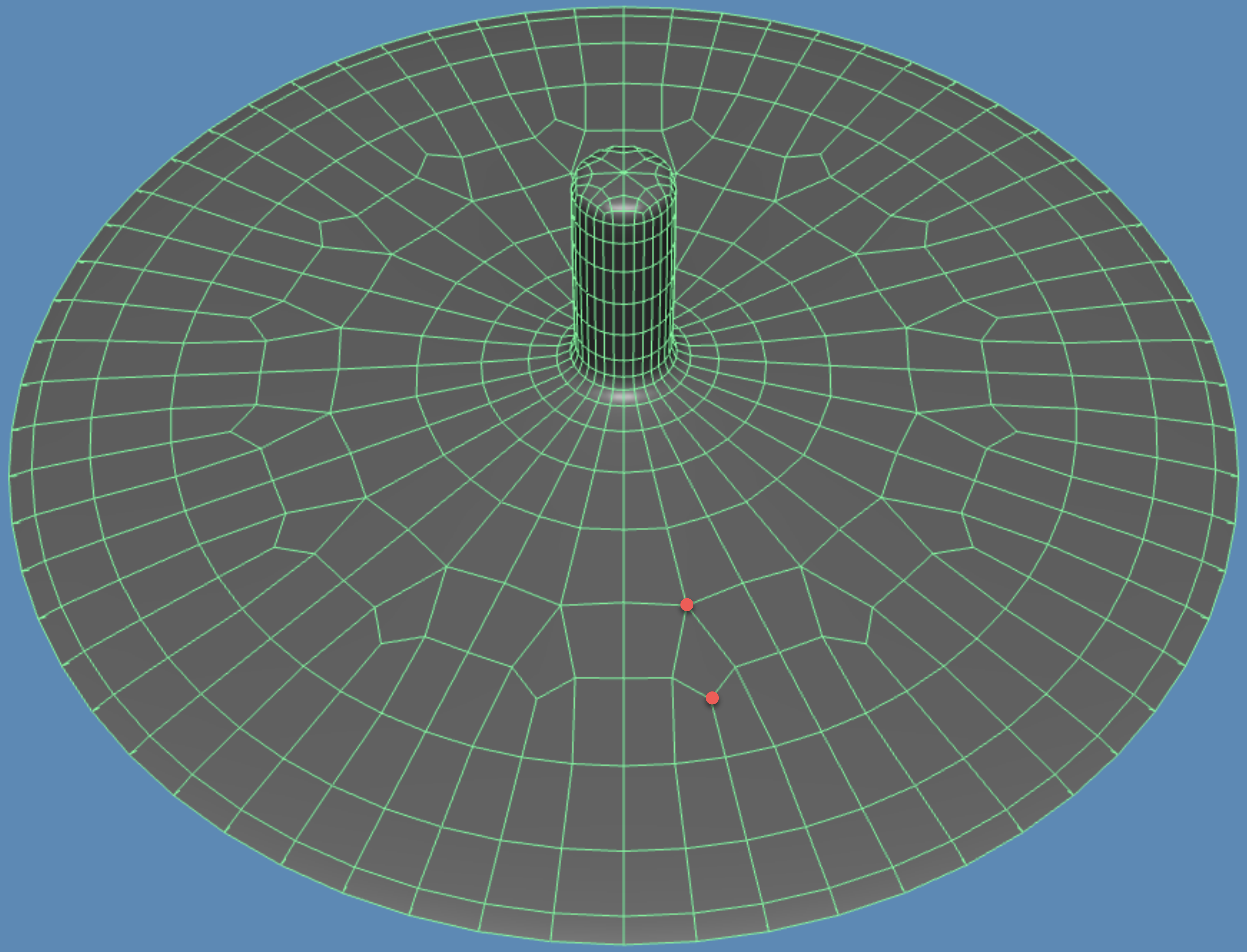

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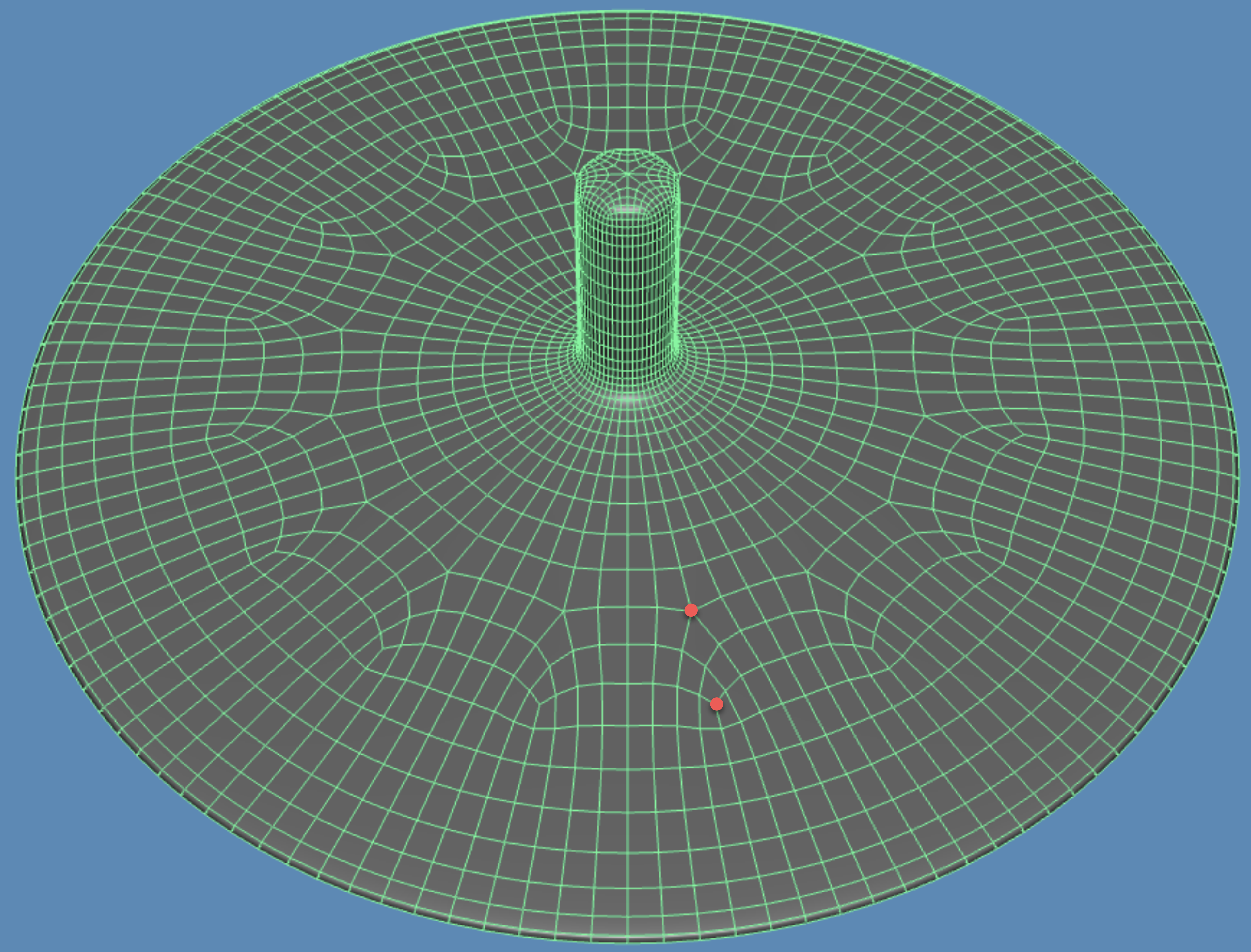

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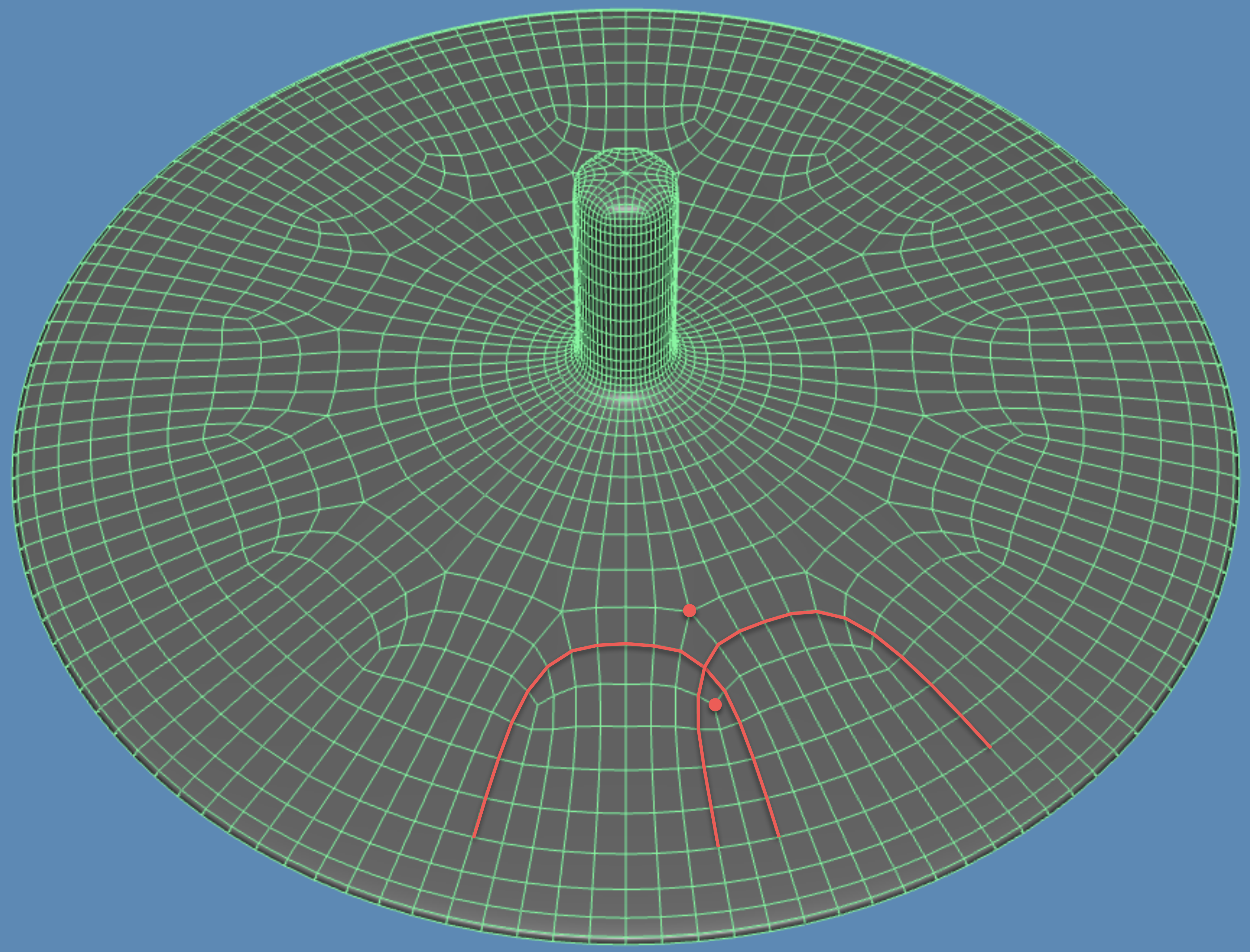

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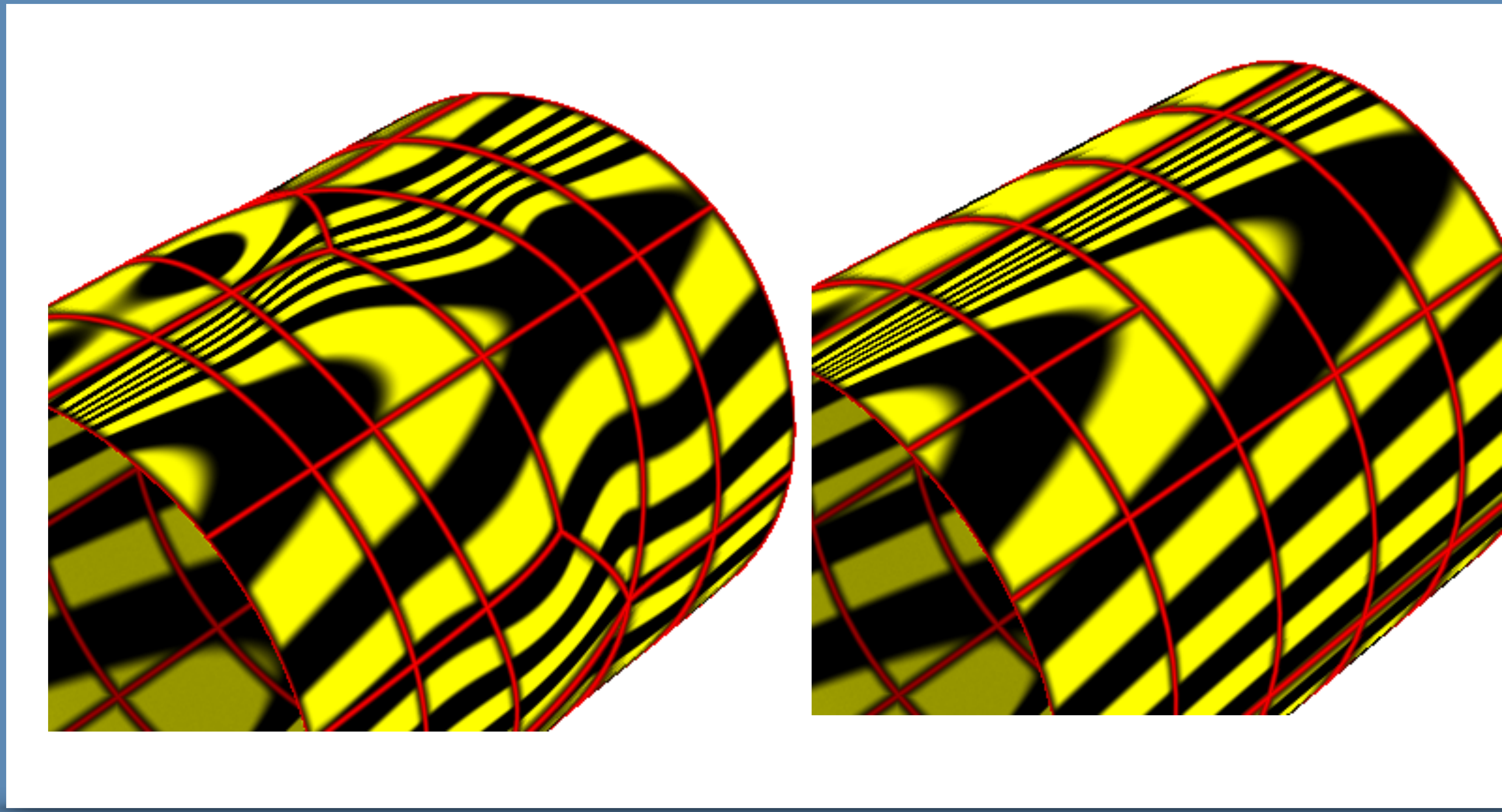
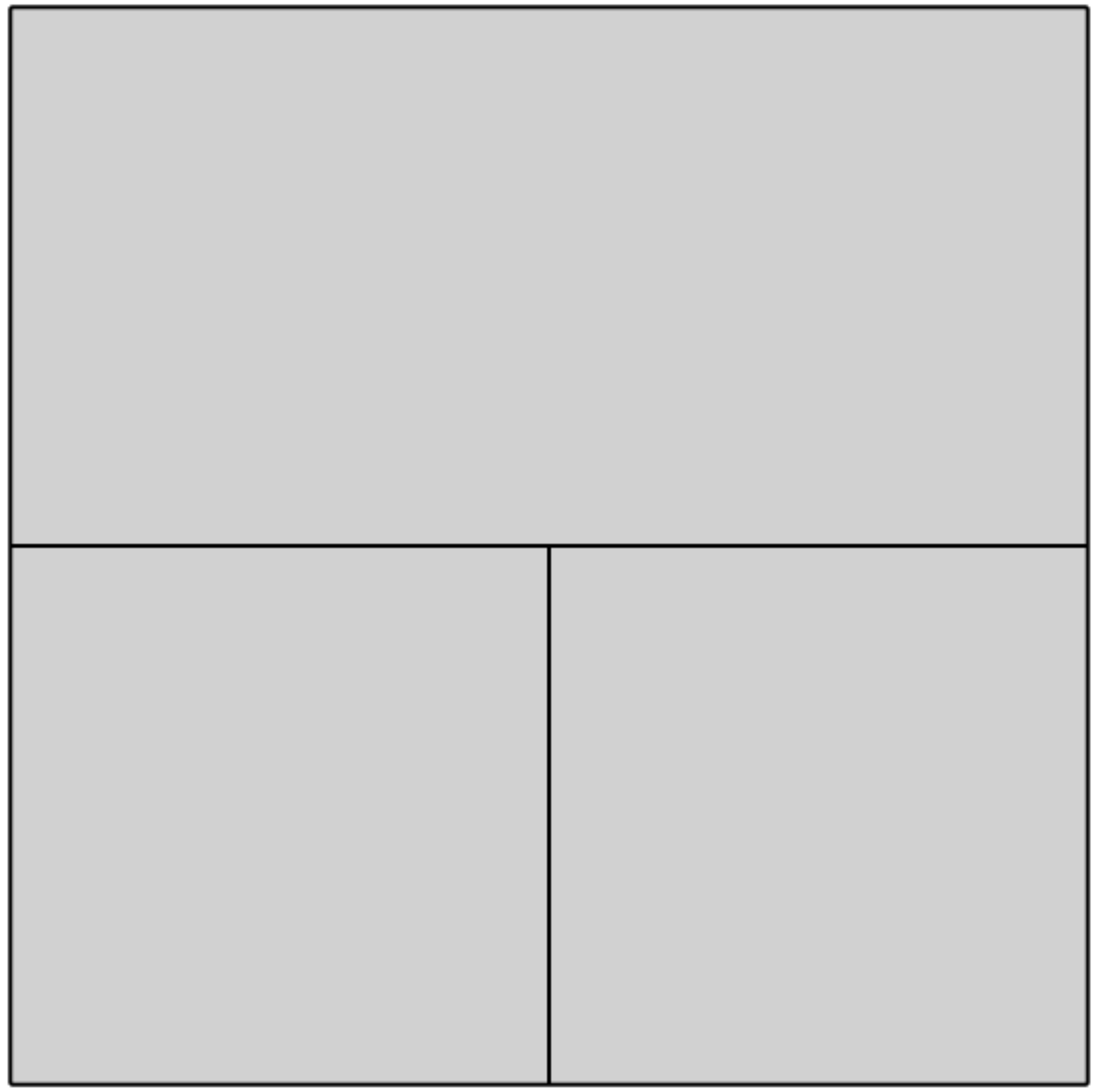
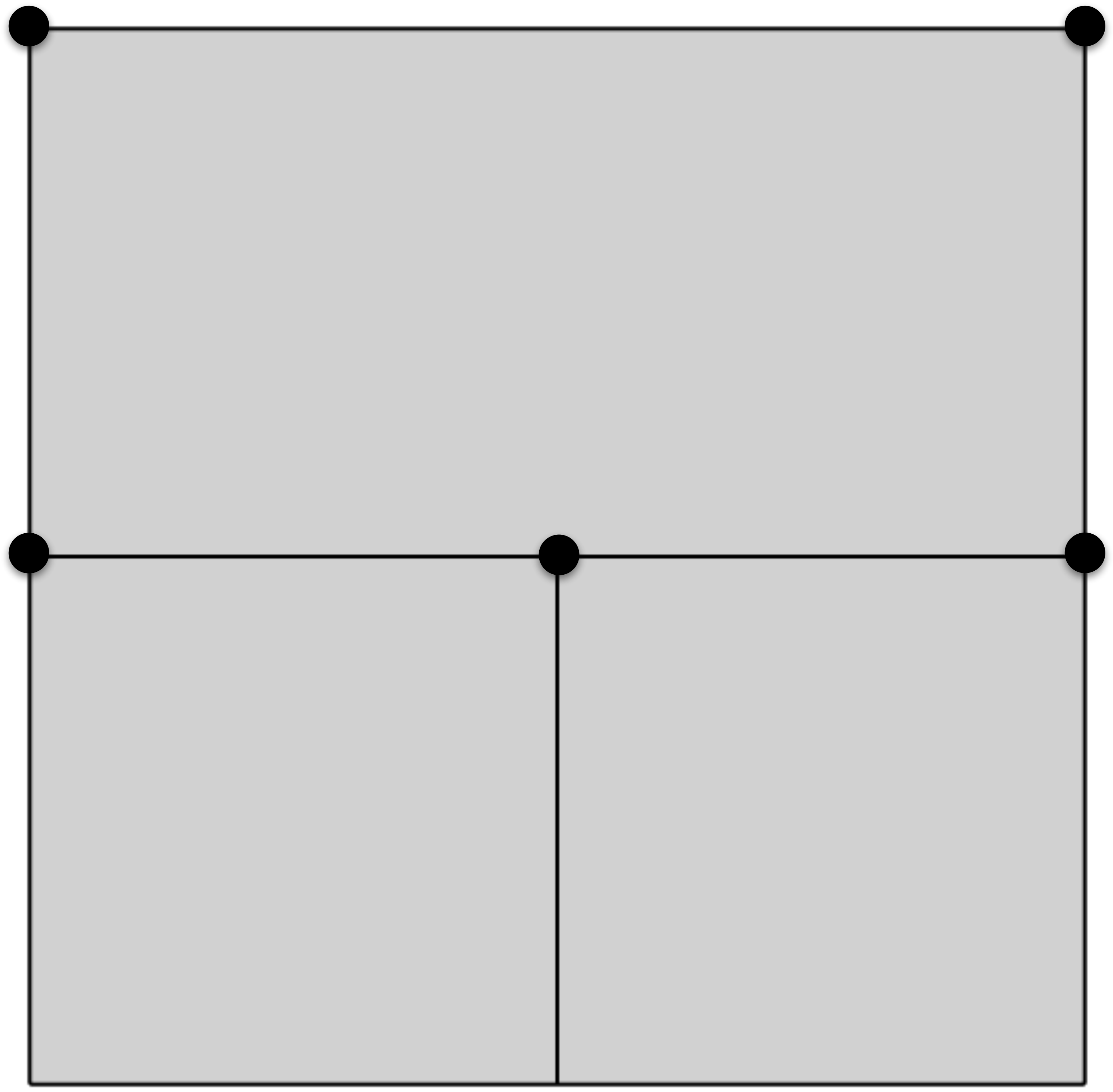
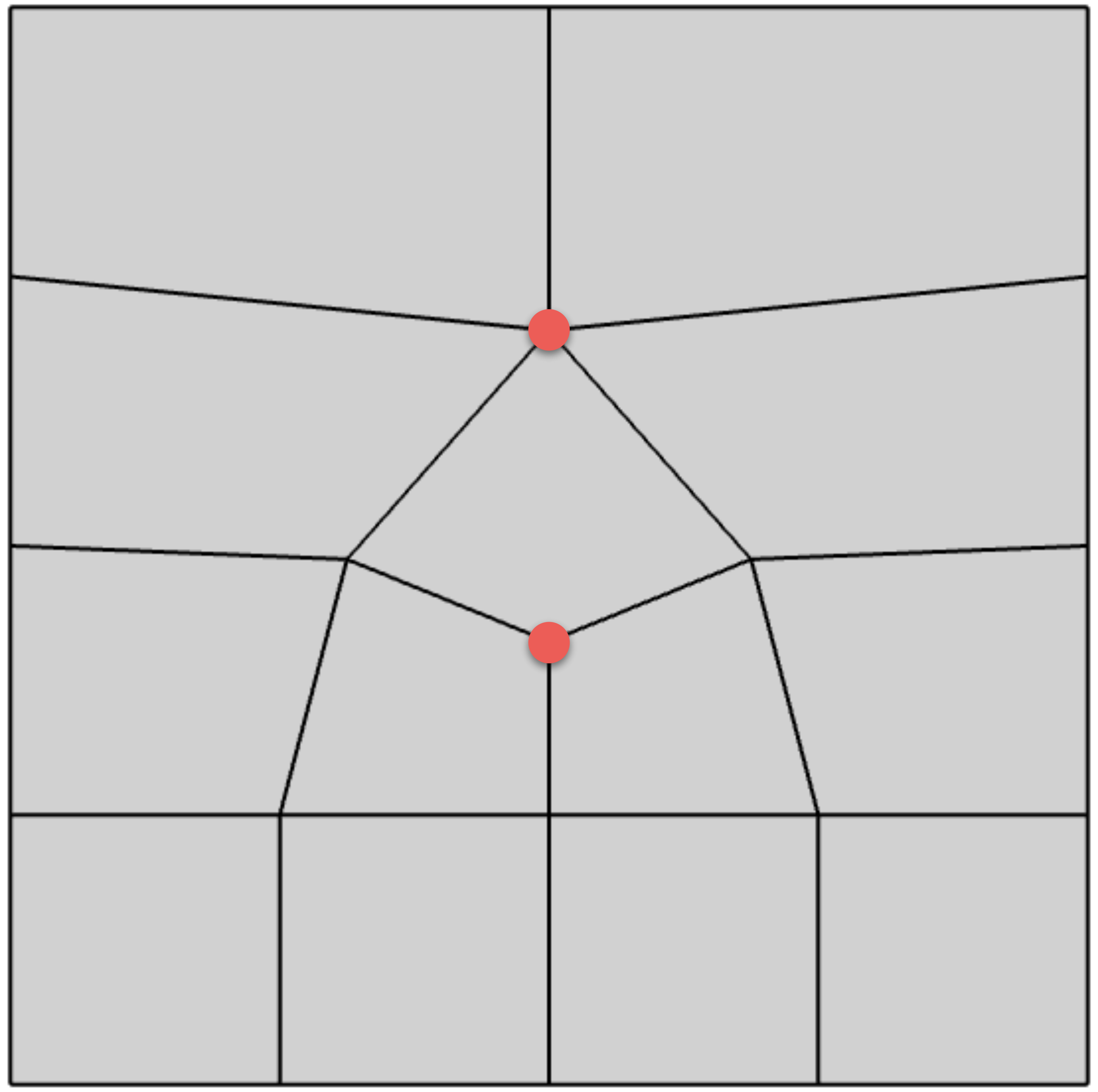
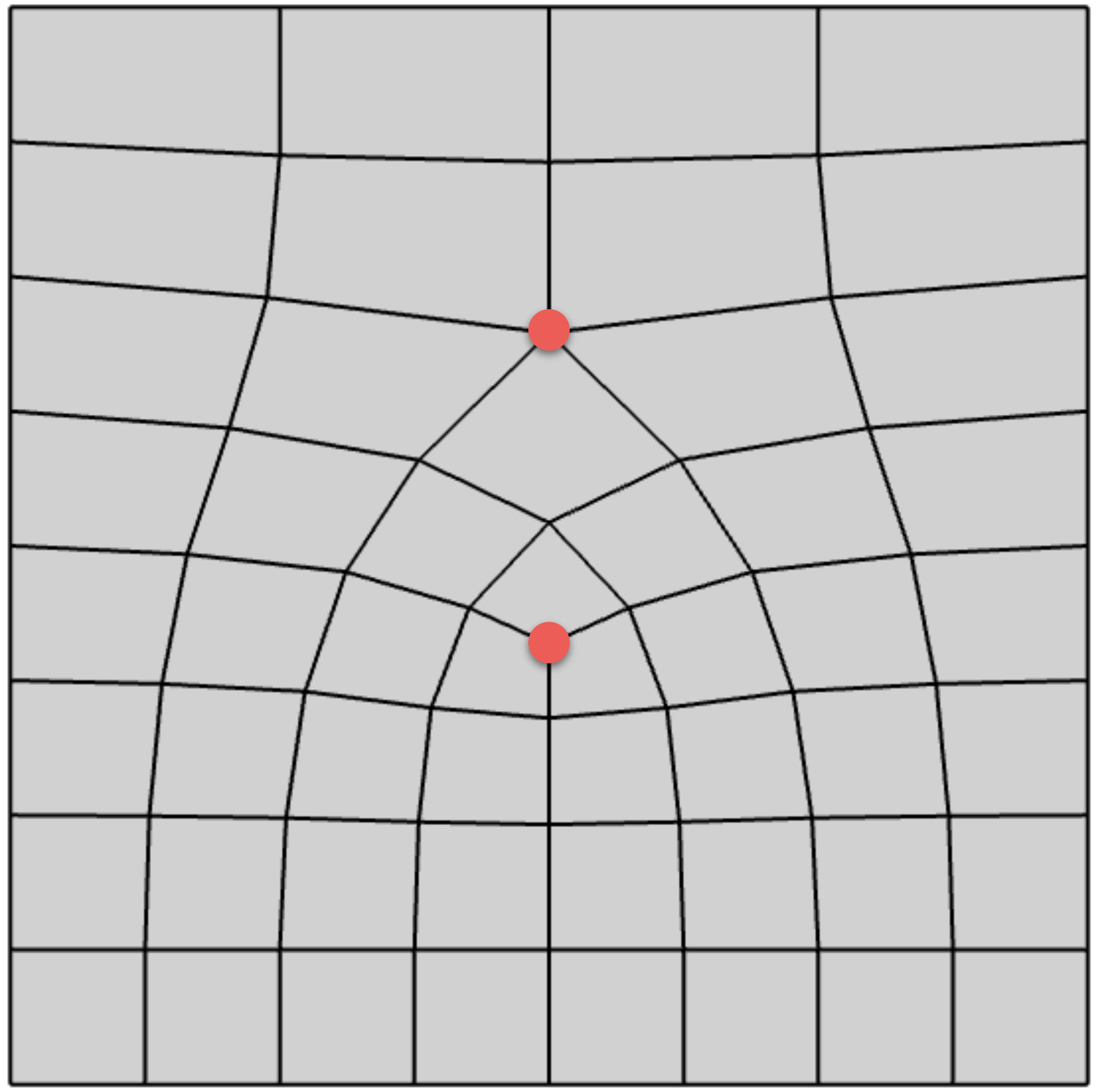
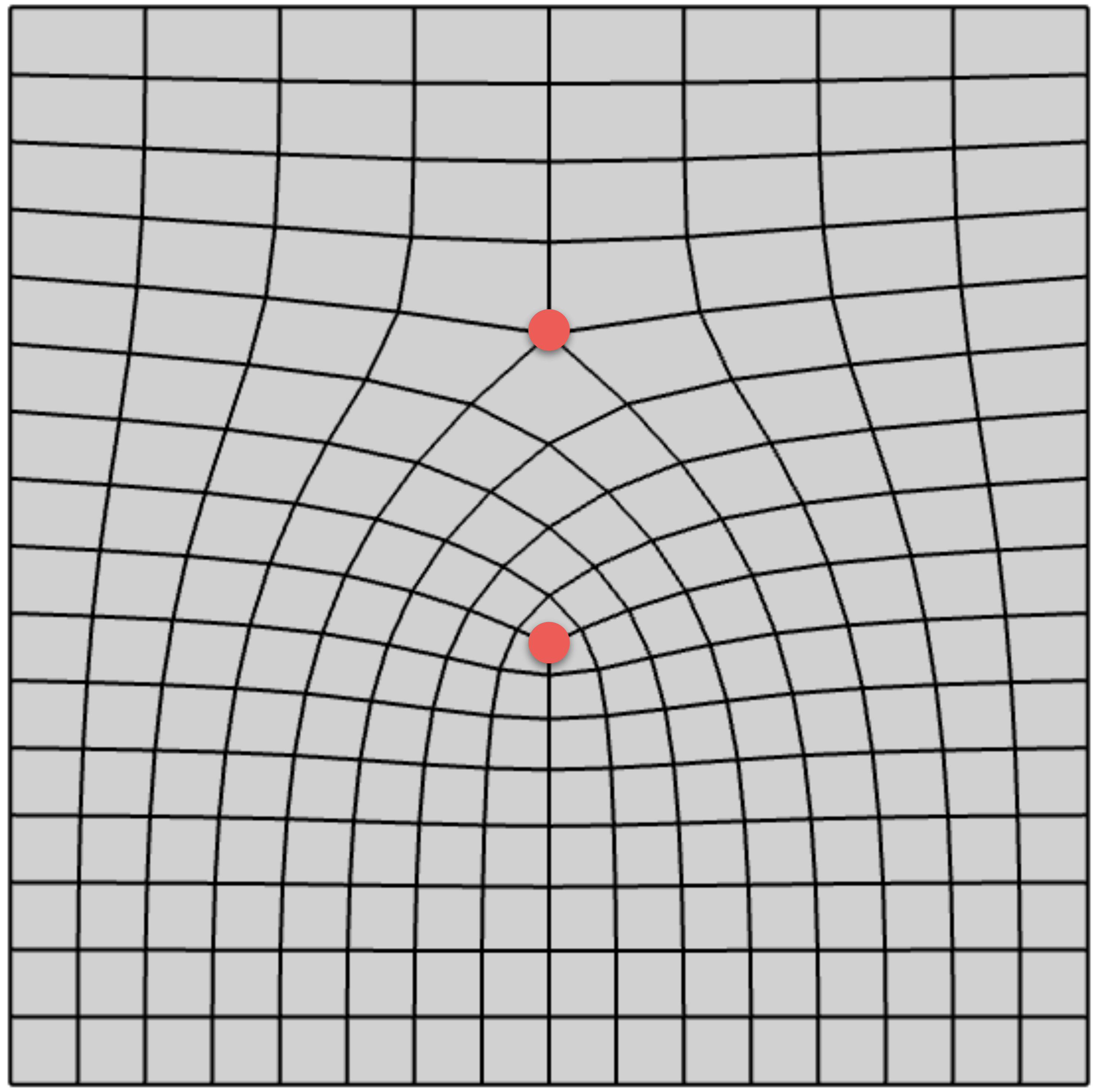


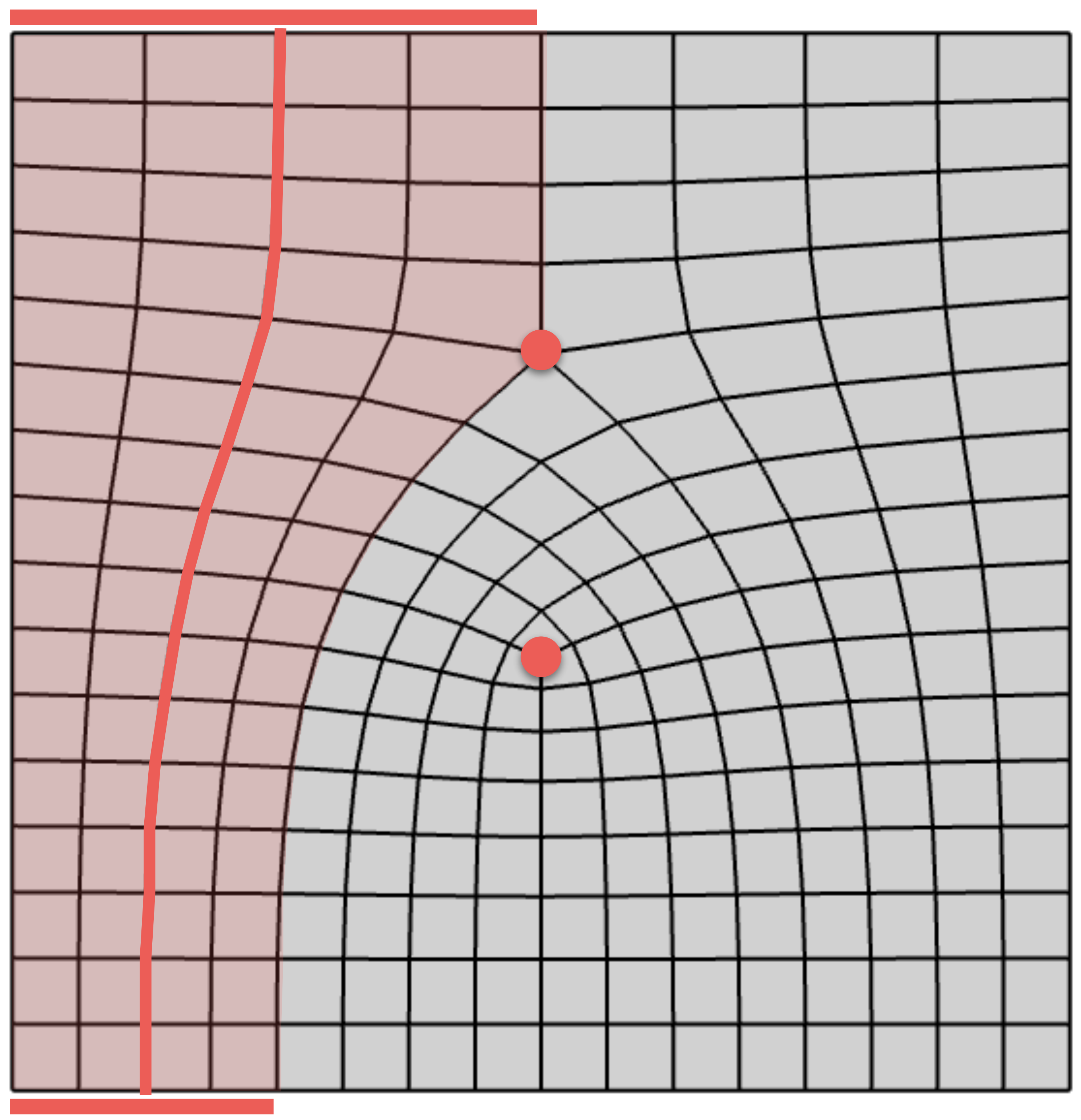


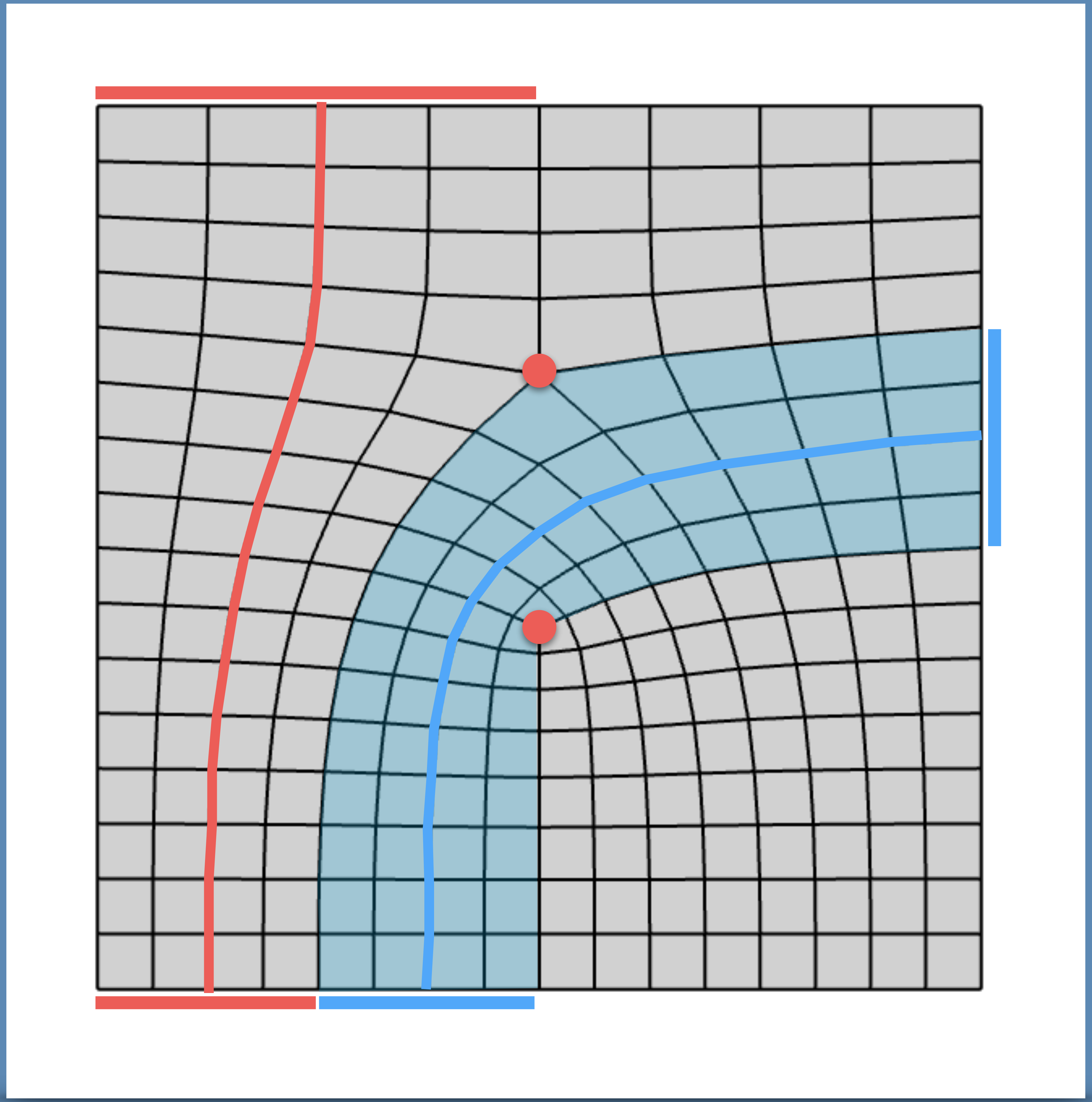


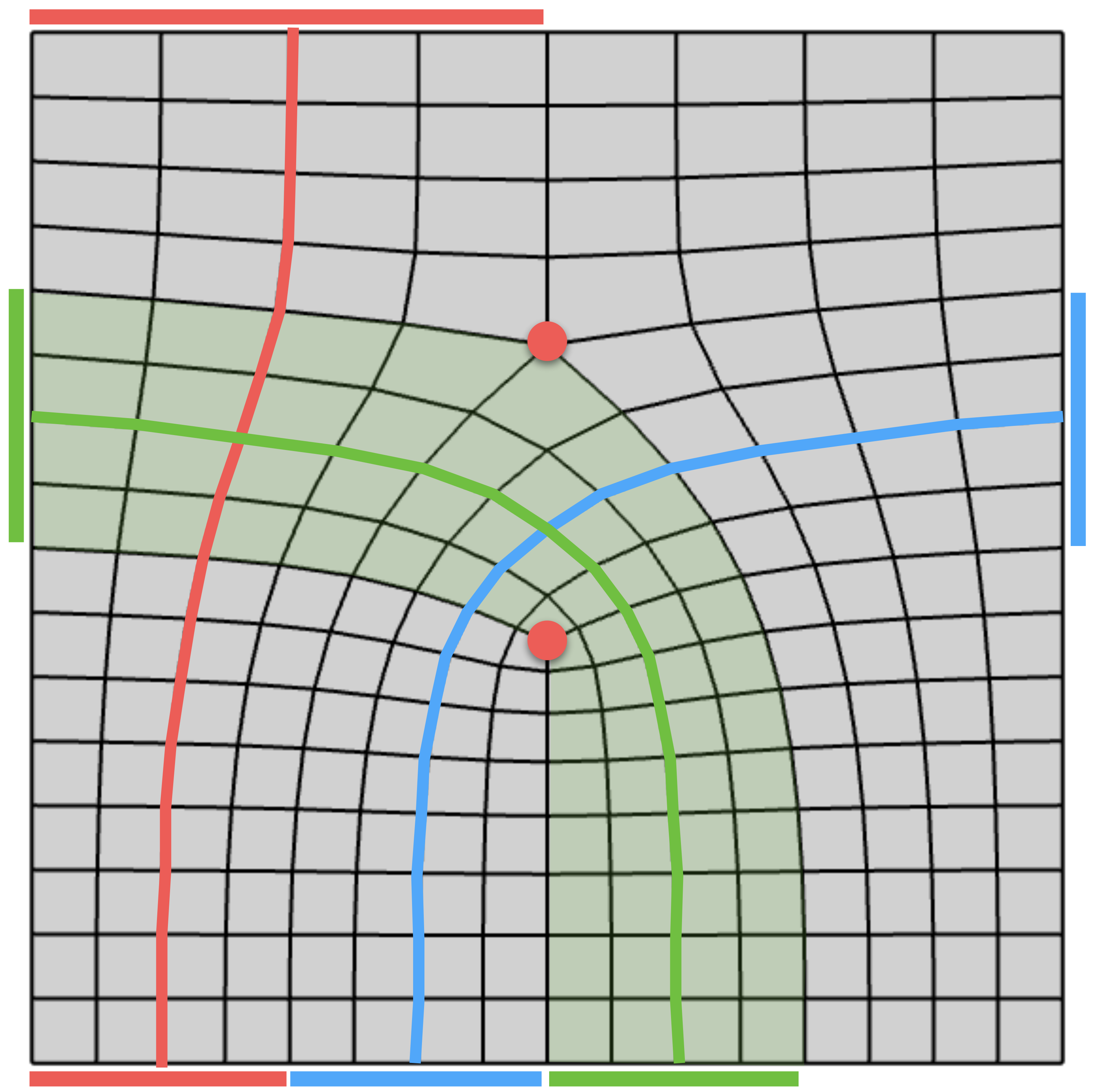


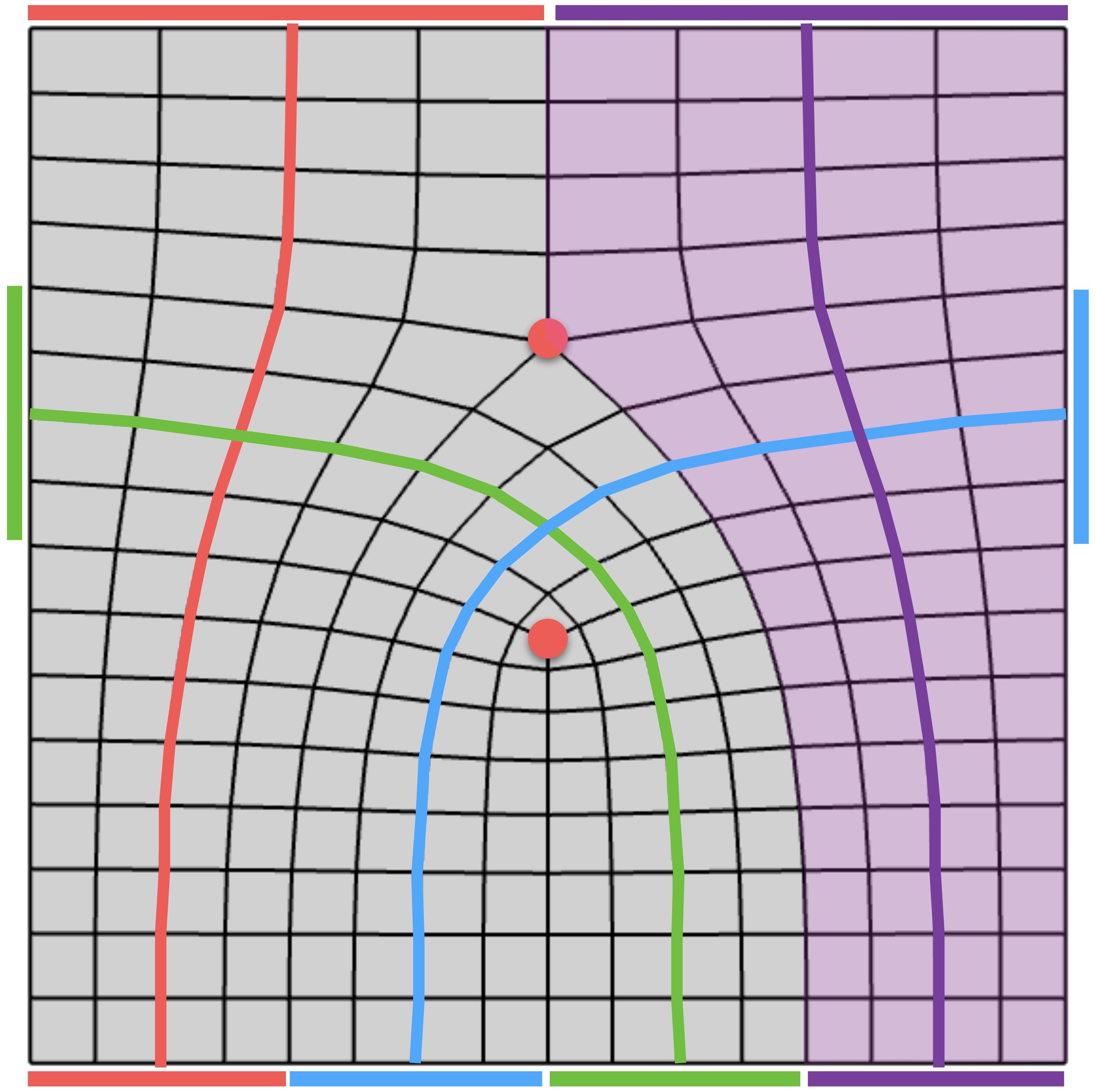


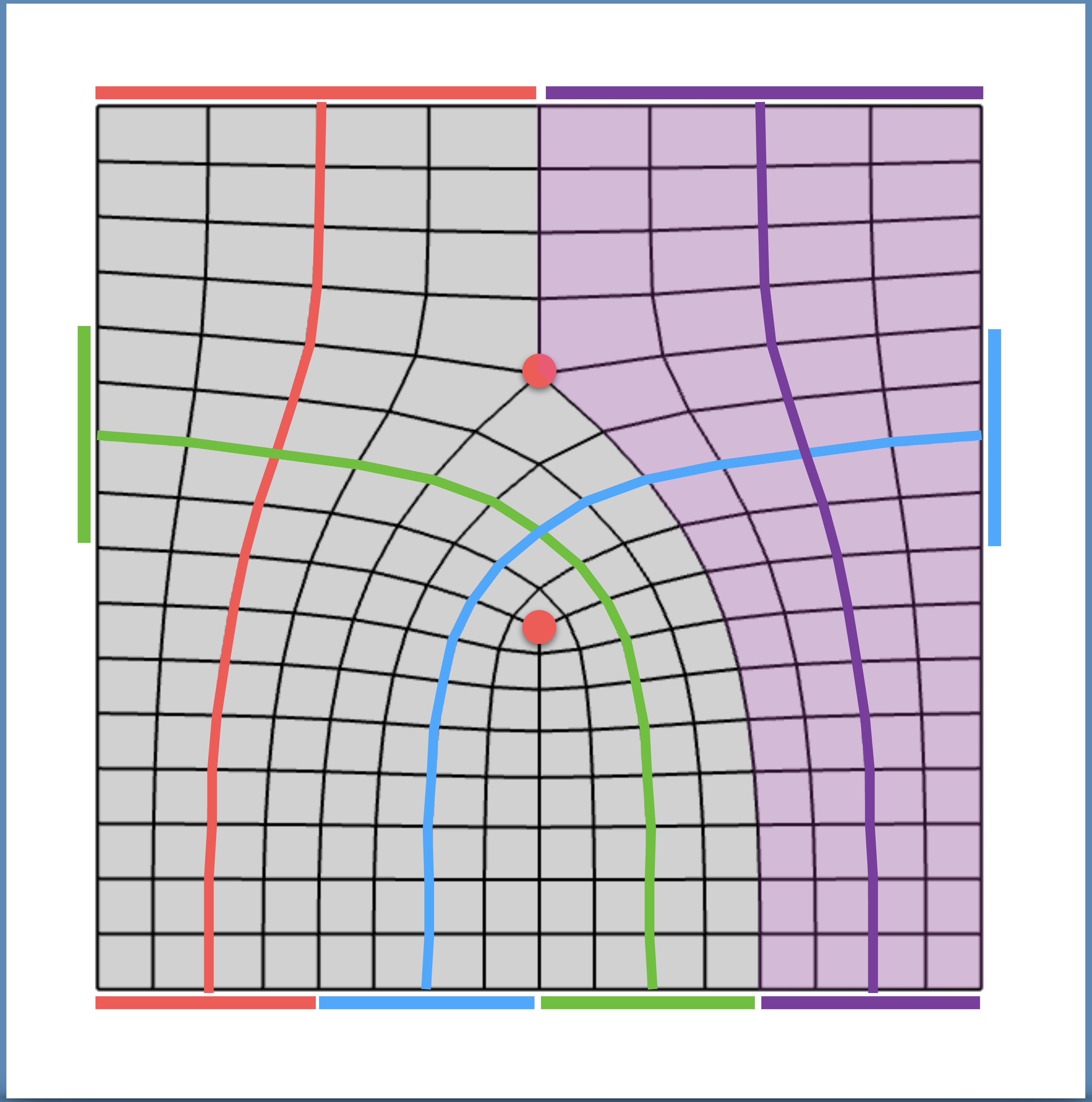




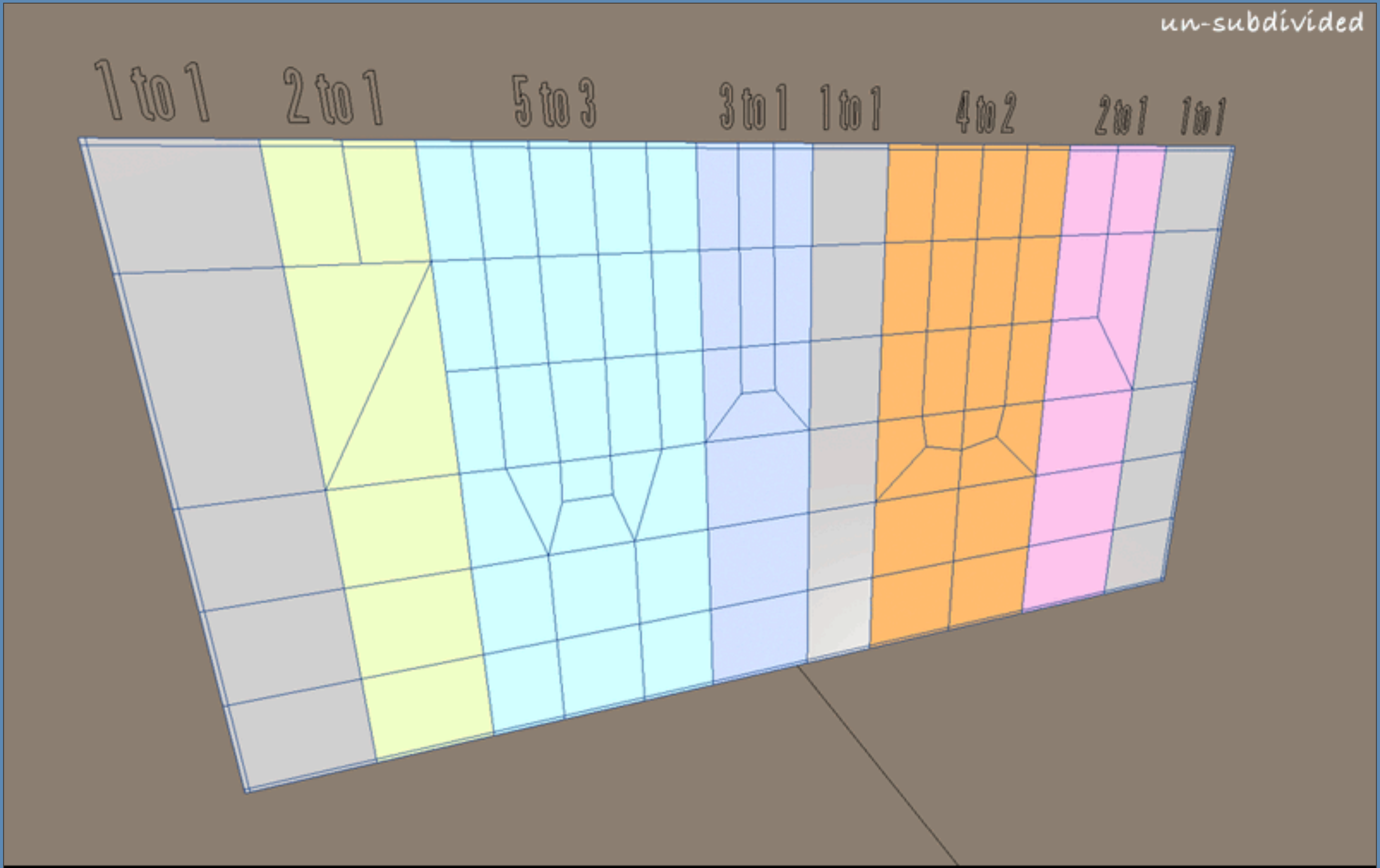






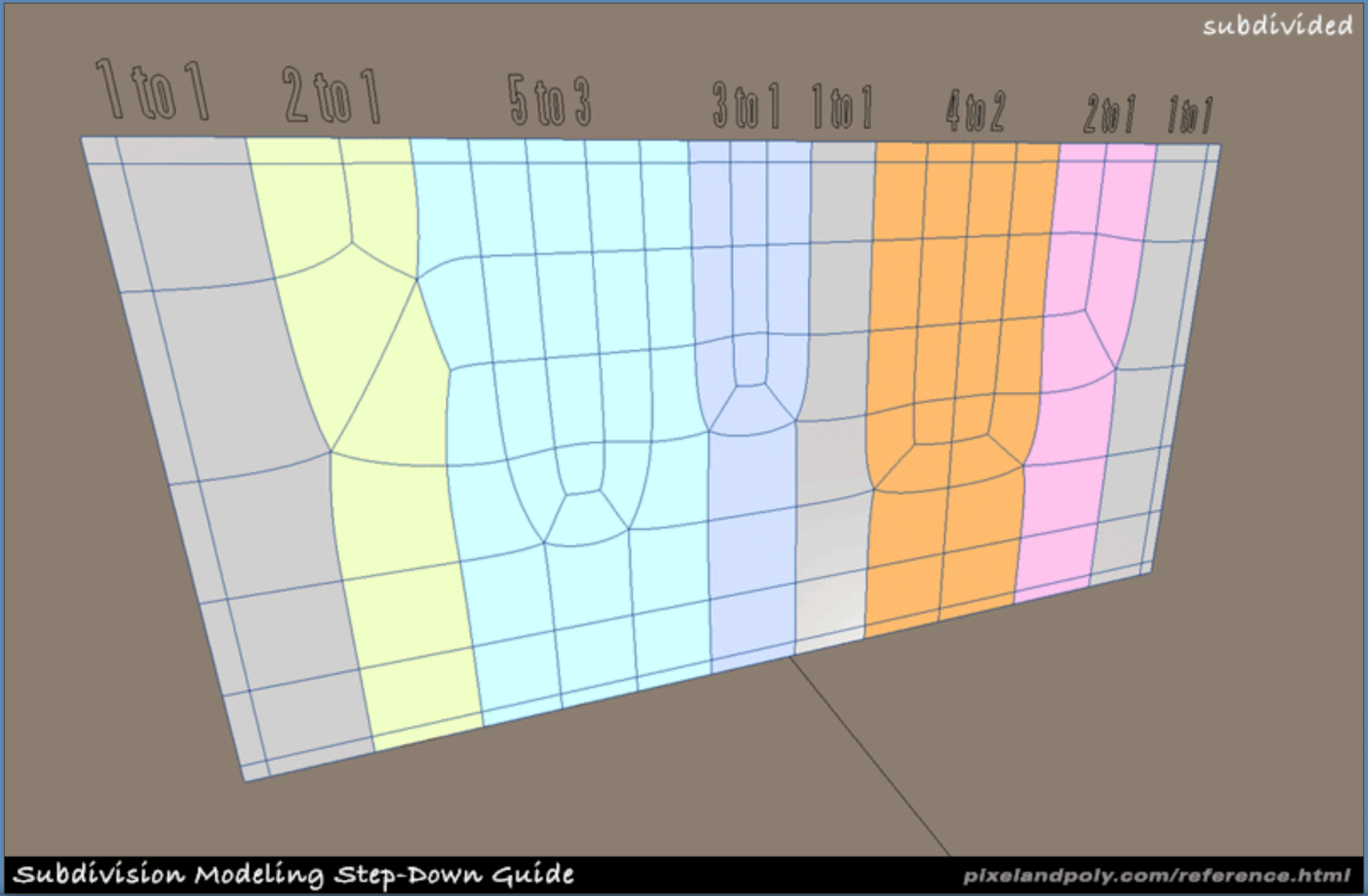


Redirected Edge Flow




Subdivision Modeling Step-Down Guide

pixelandpoly.com/reference.html


subdivided
1 to 1
2 to 1
5 to 3
3 to 1
1 to 1
4 to 2
2 to 1
1 to 1
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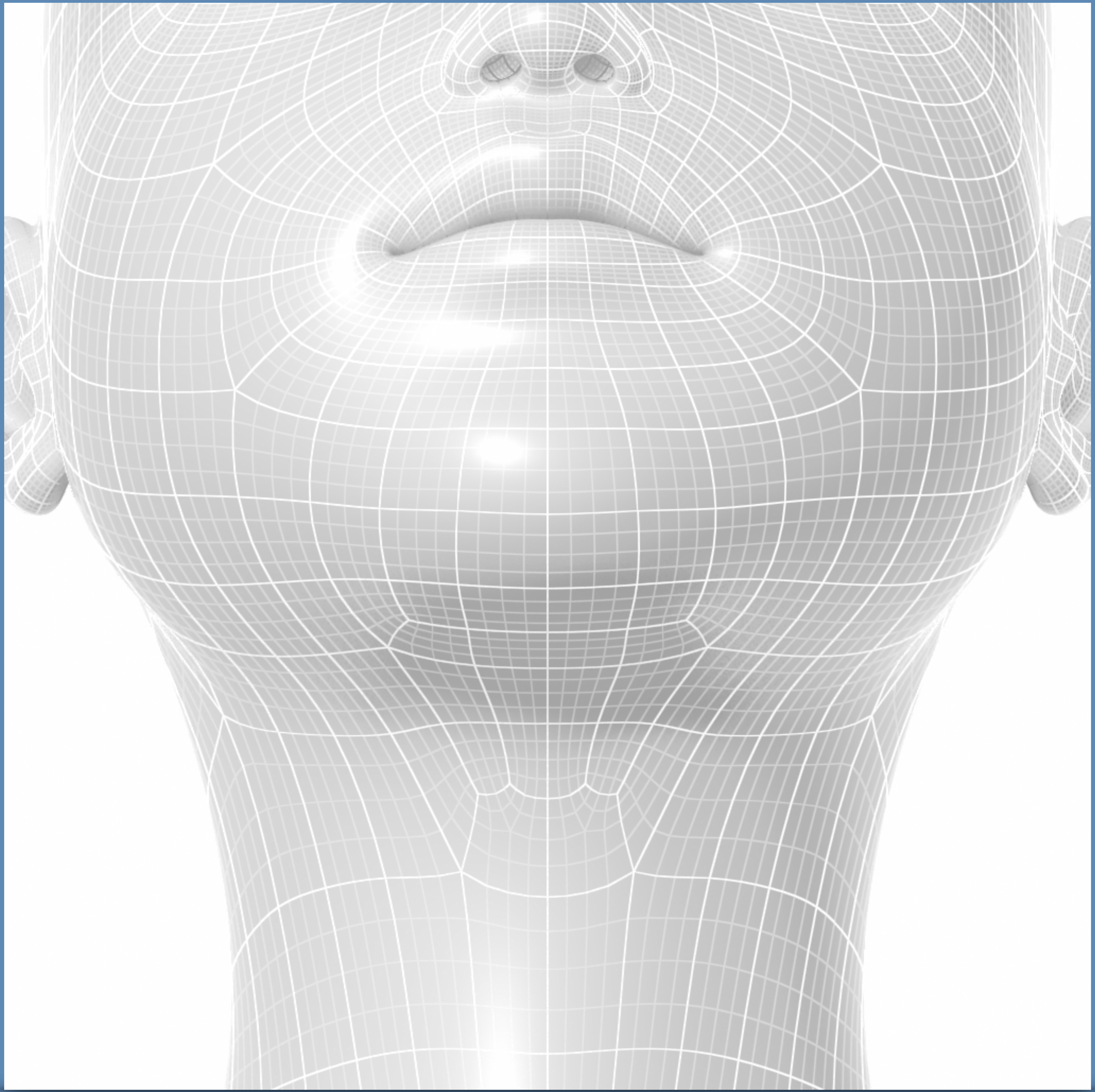

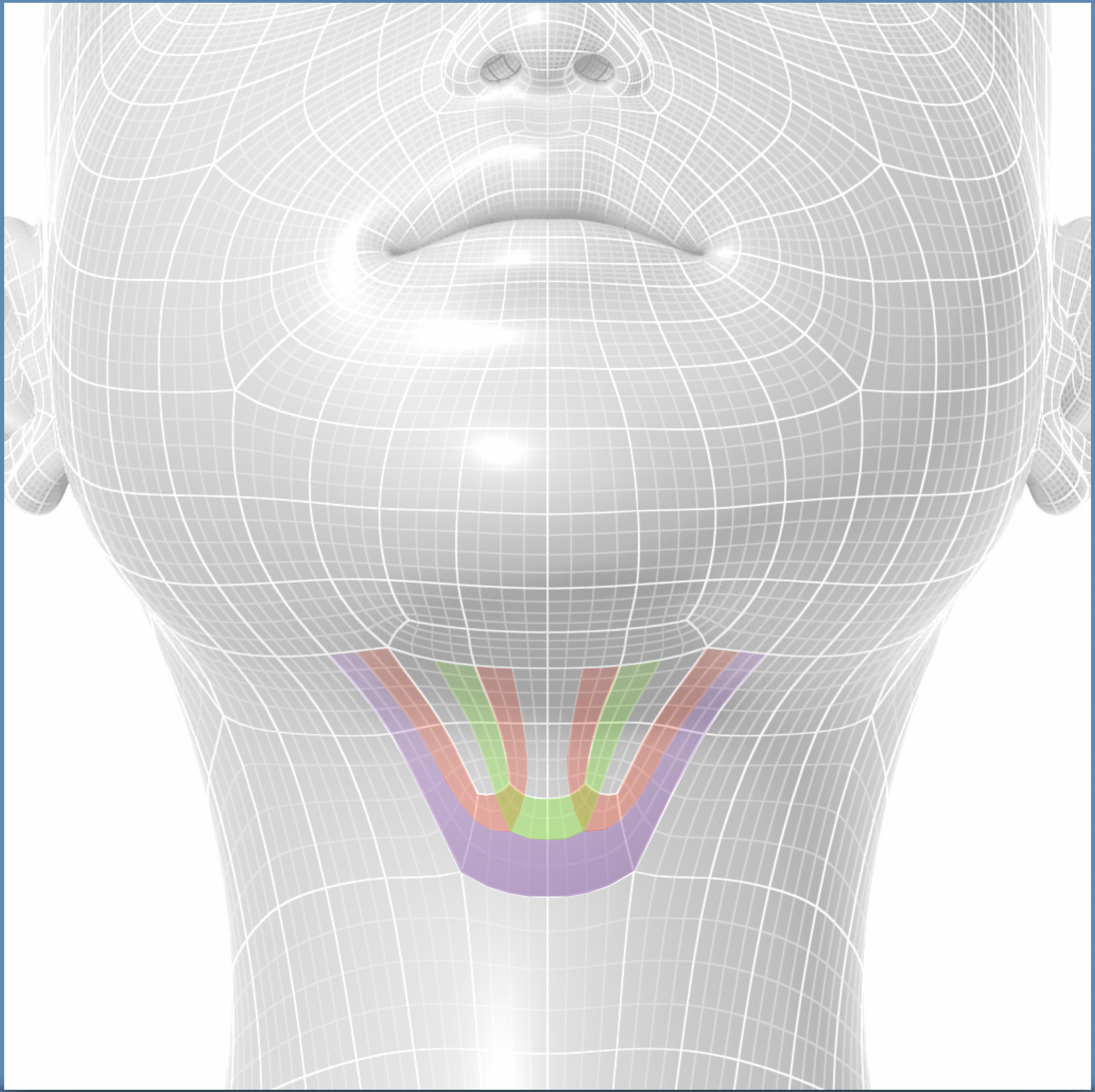

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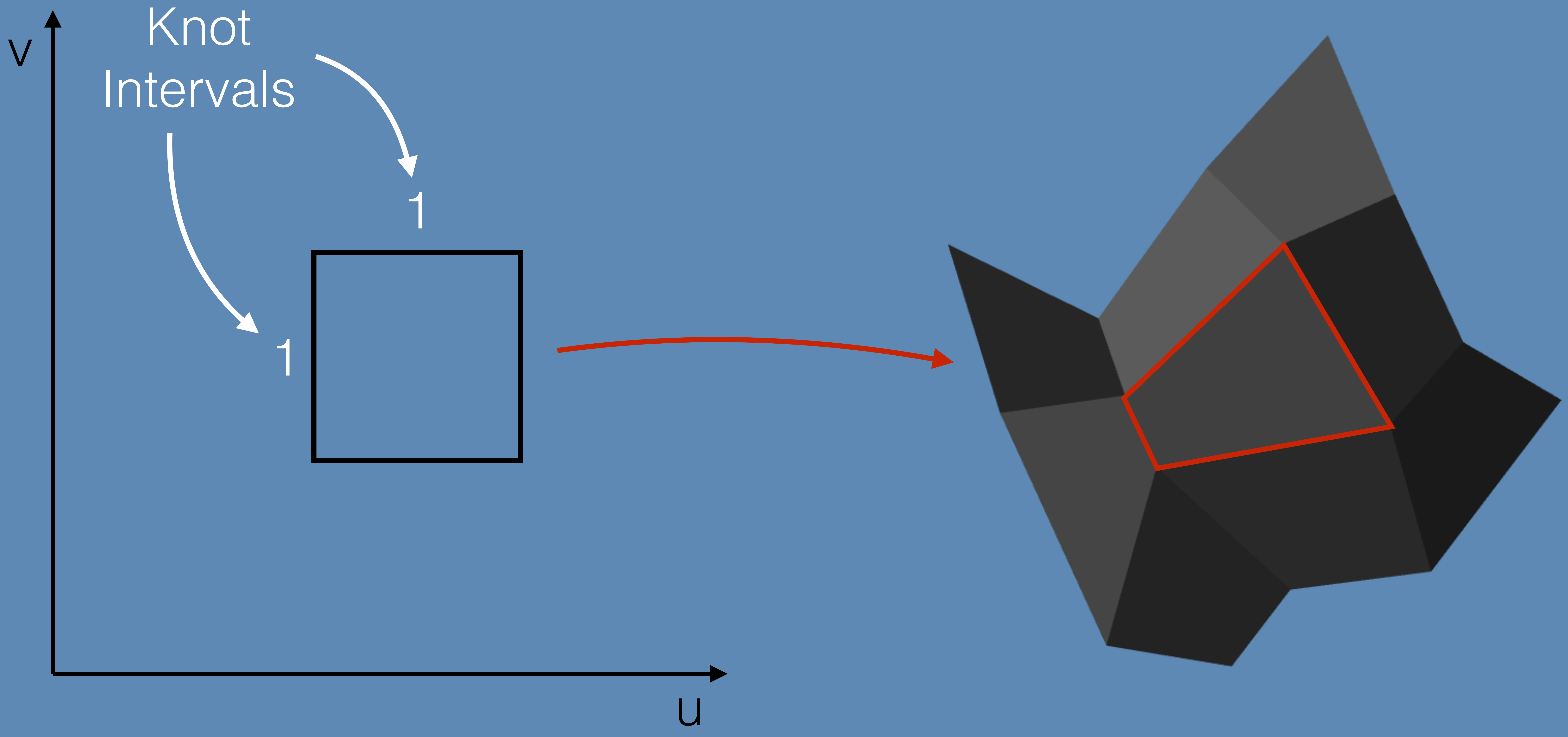

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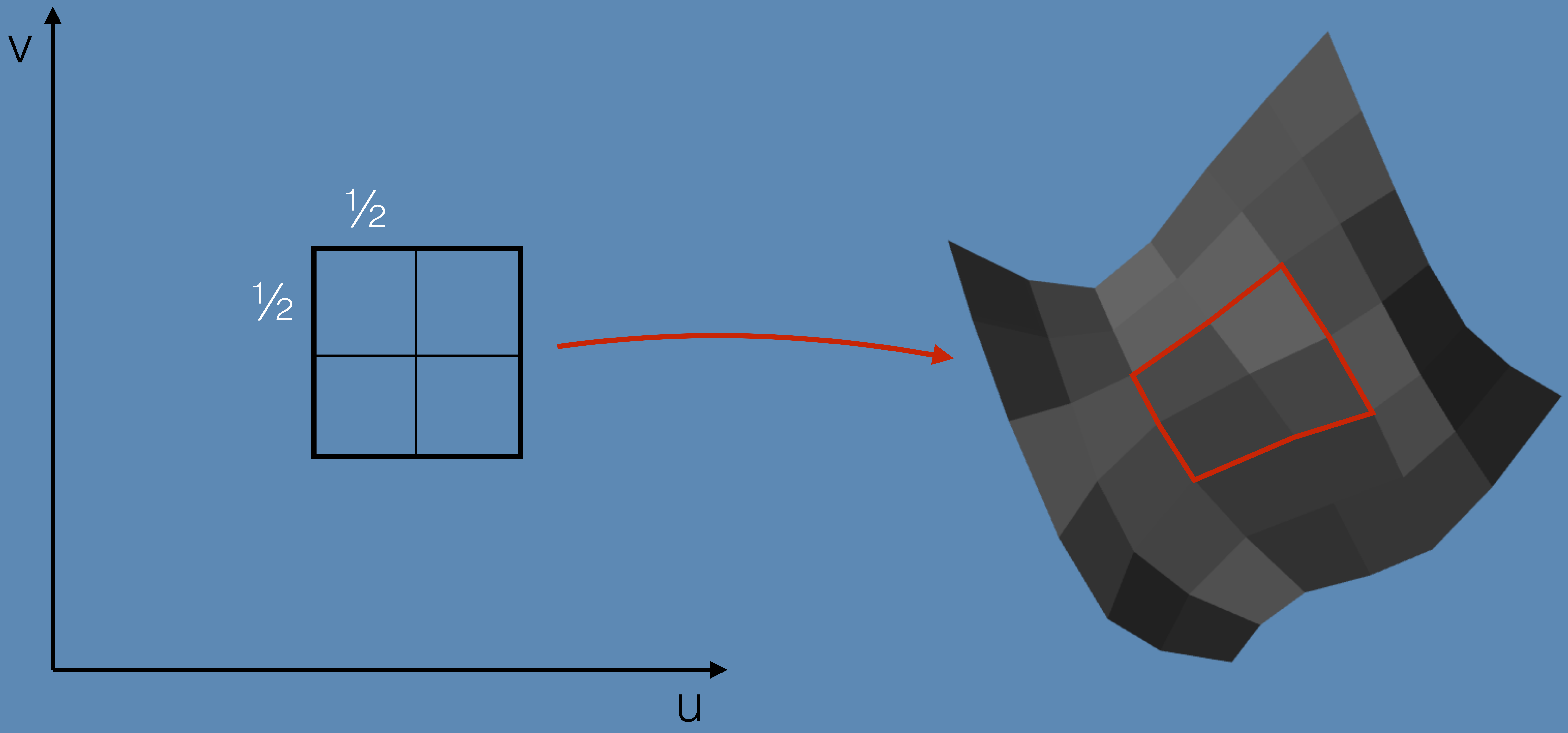

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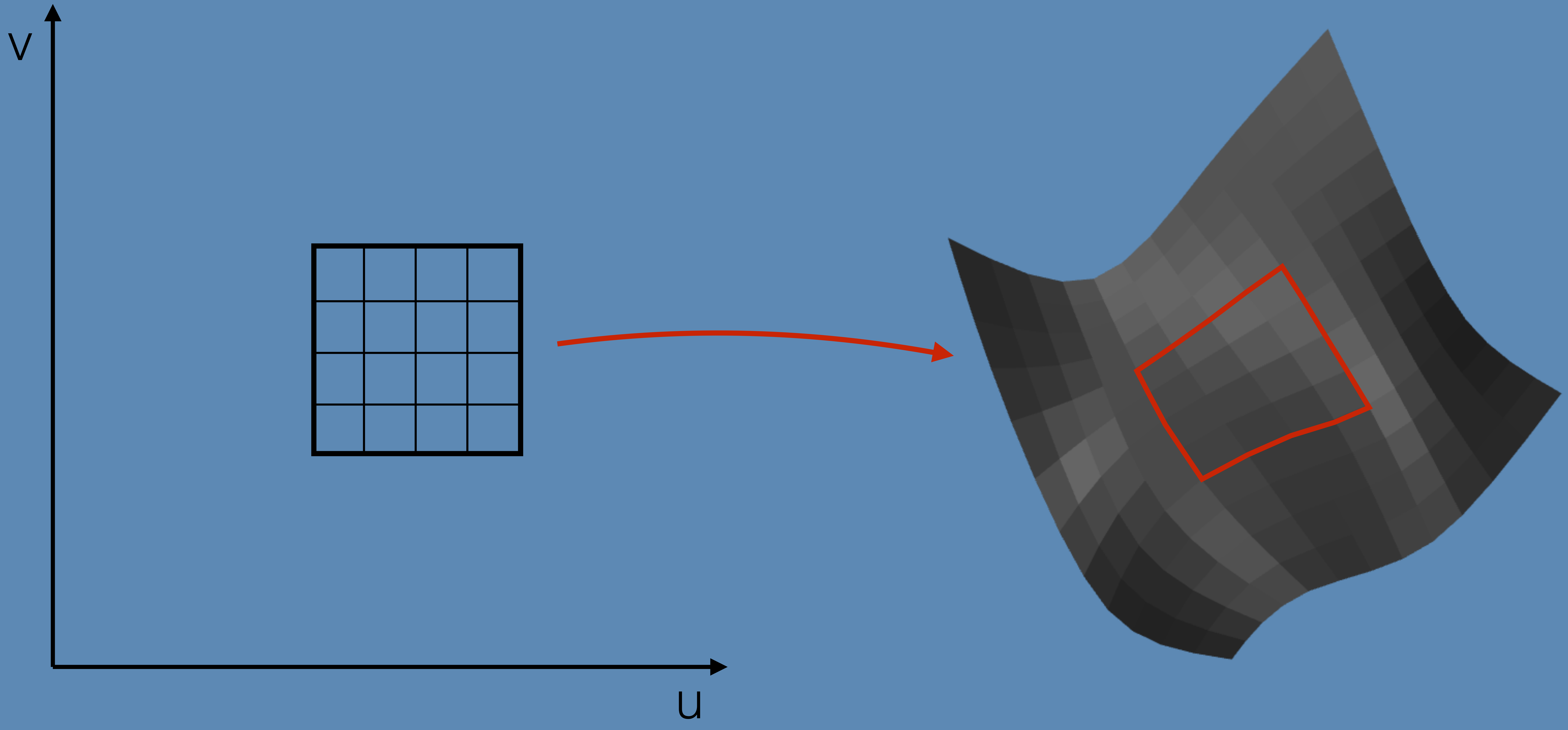

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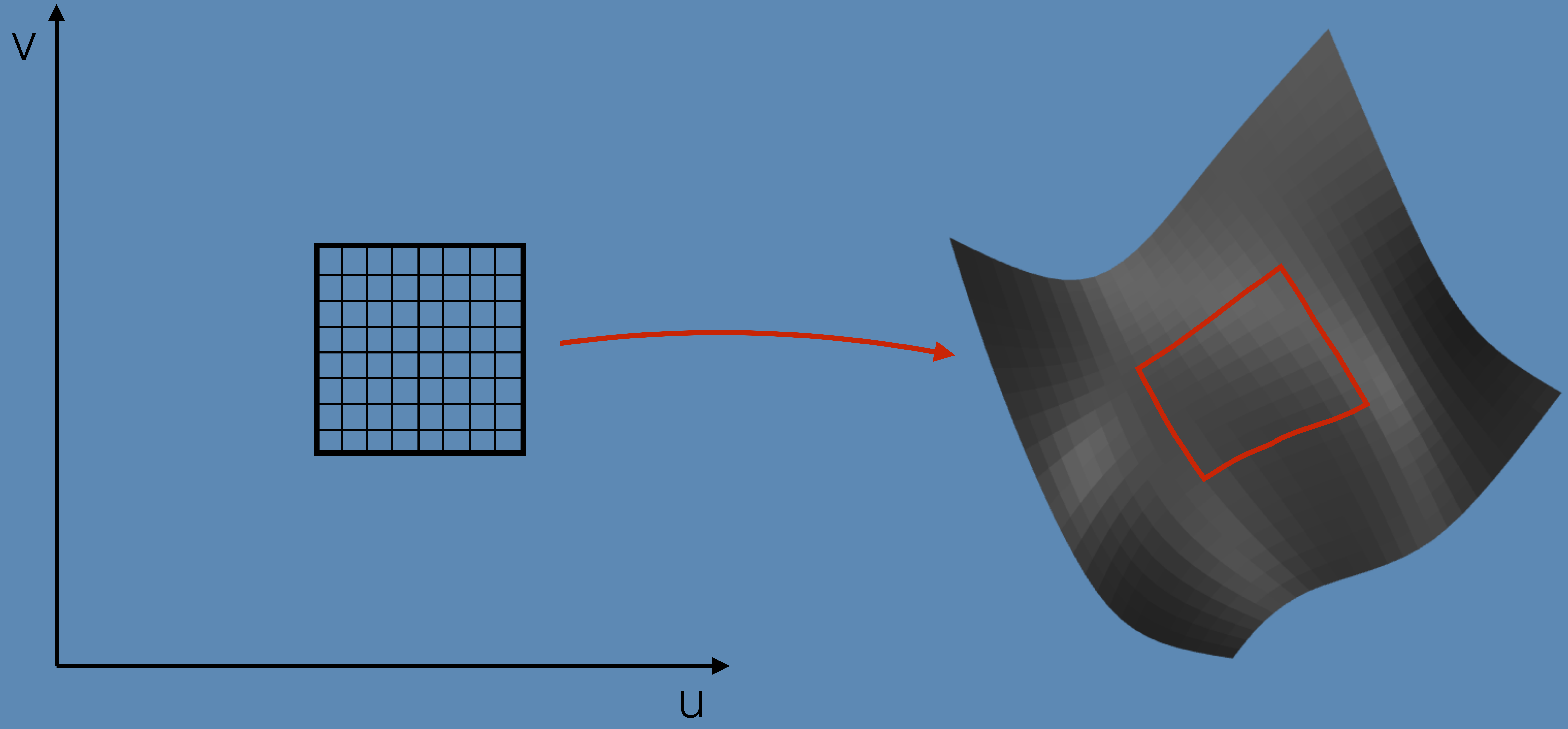

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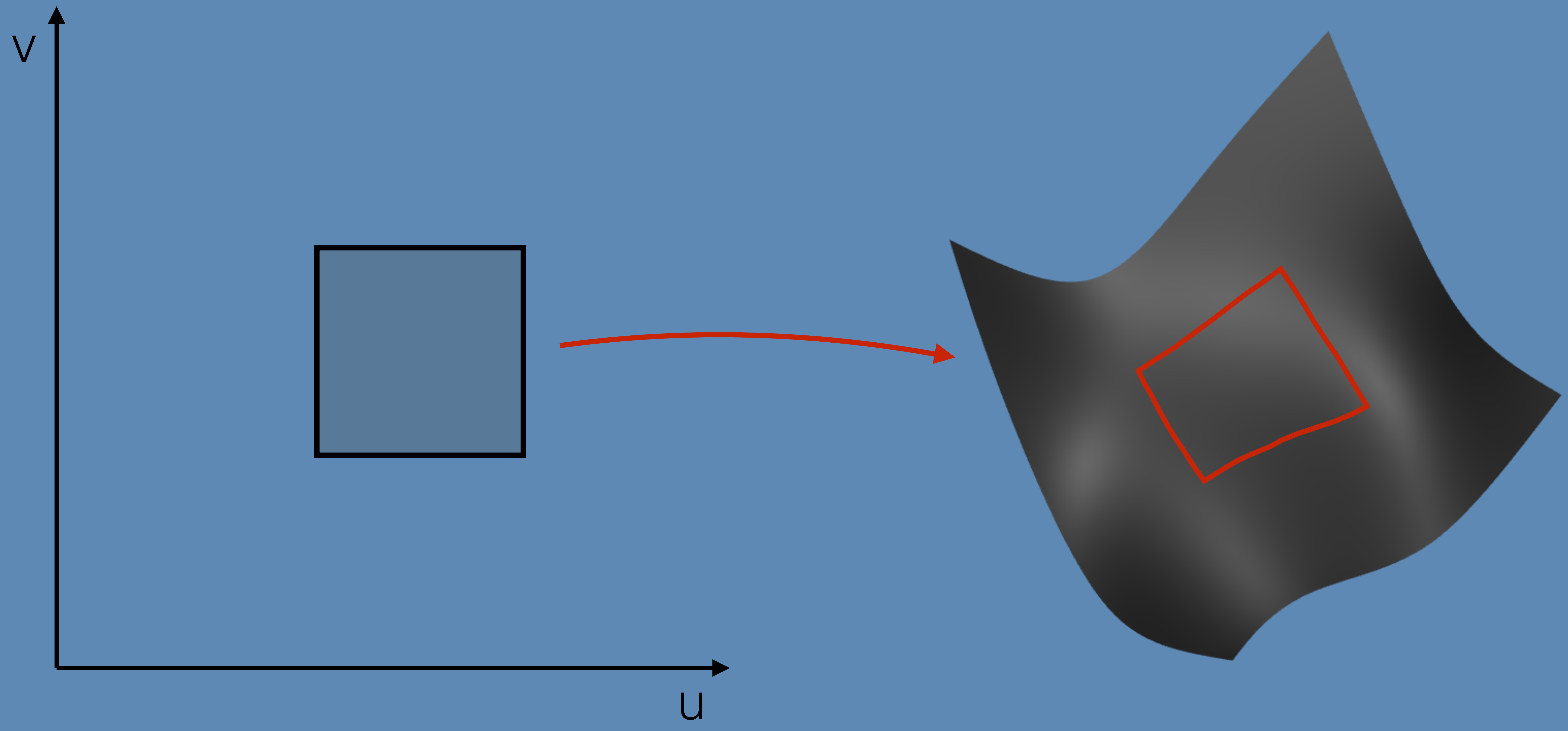
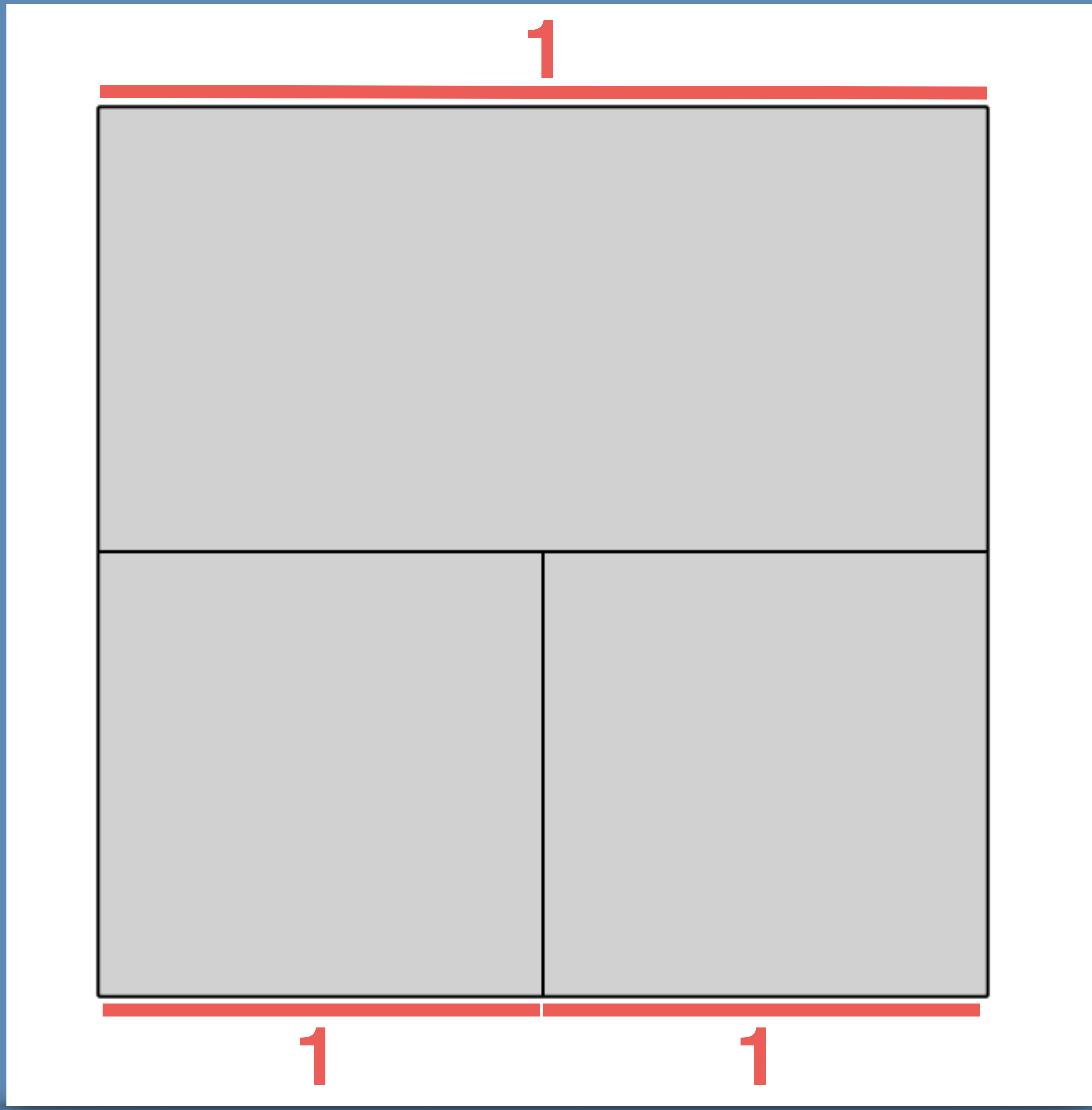
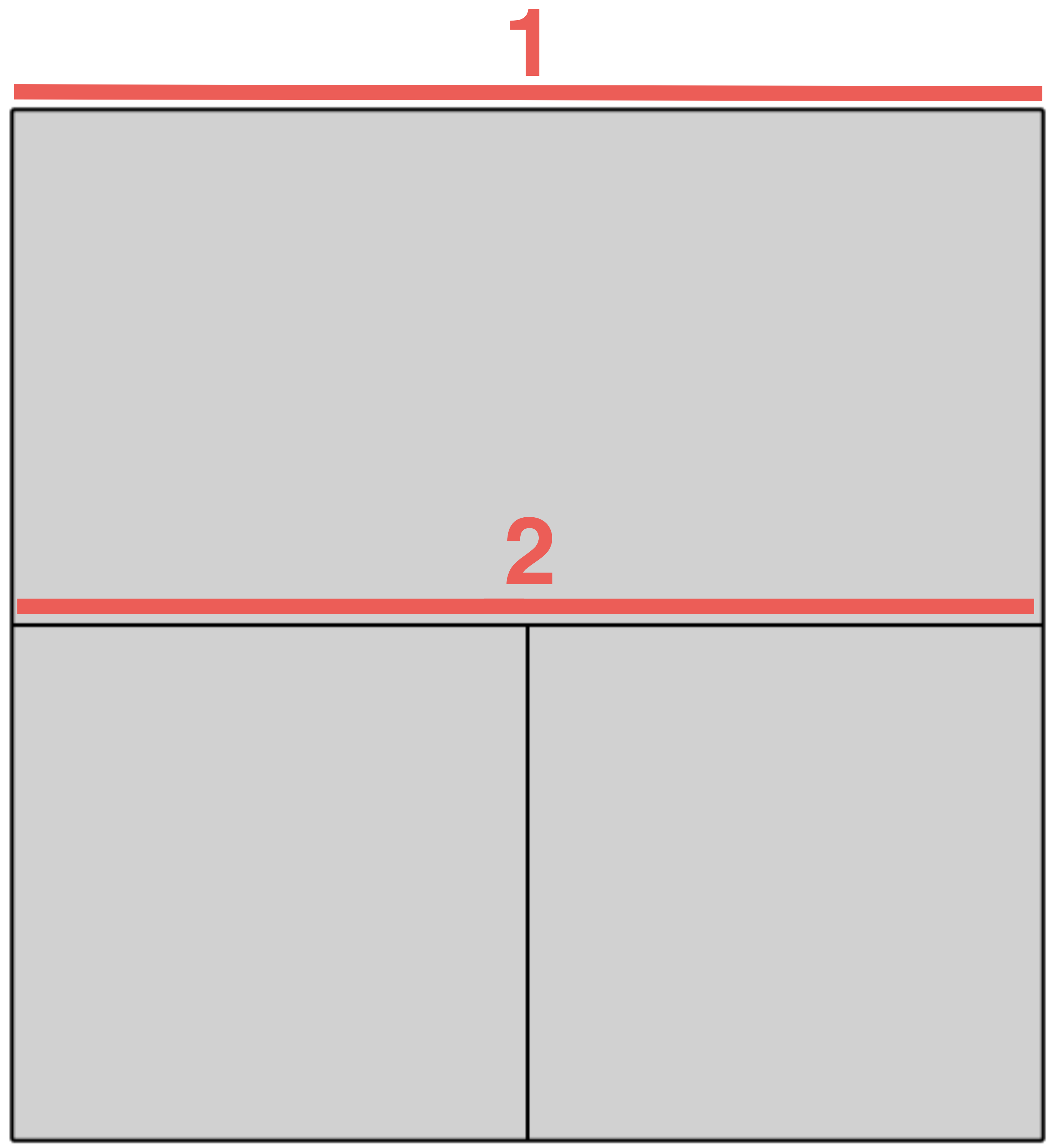





uniform knot intervals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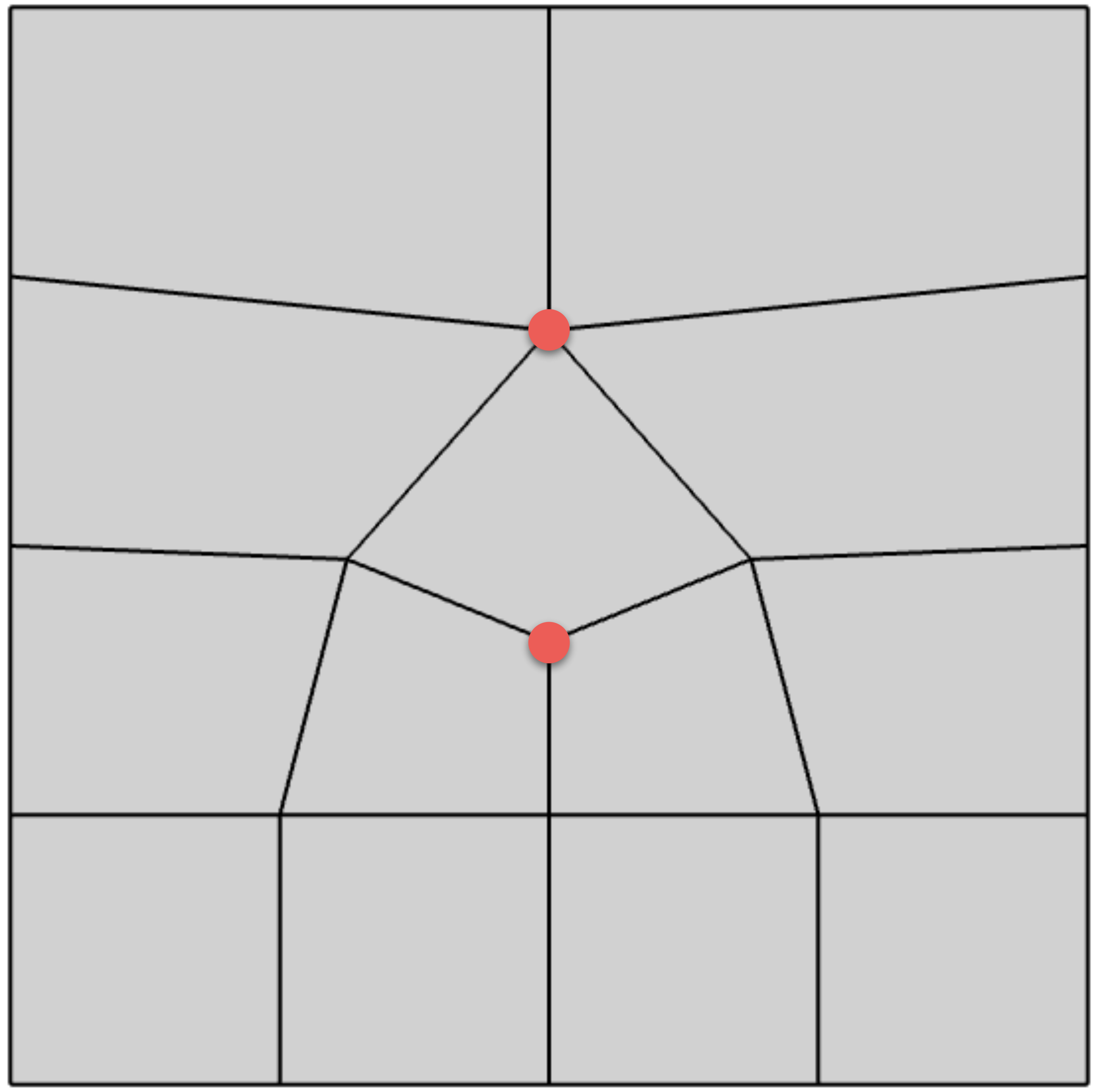

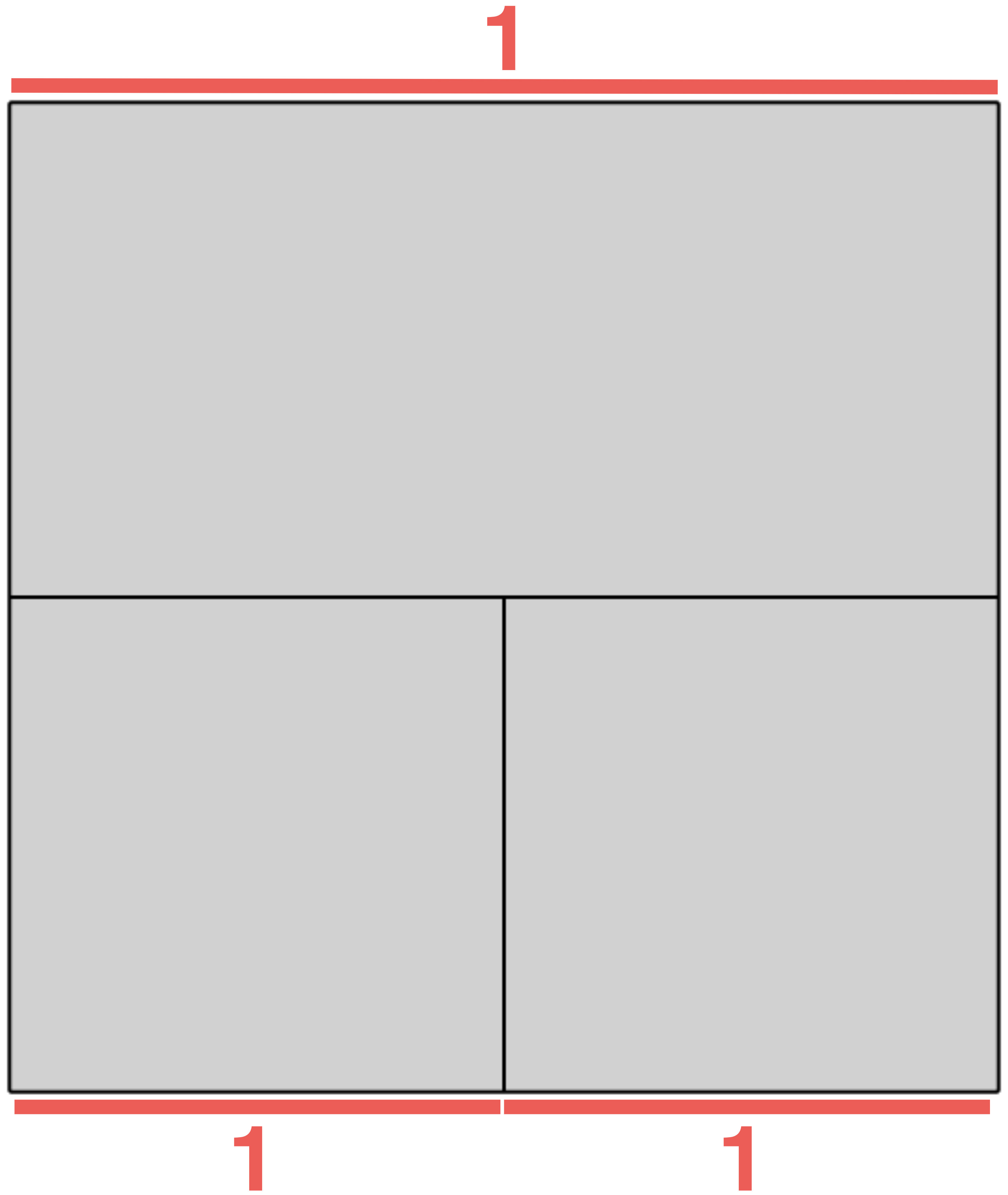
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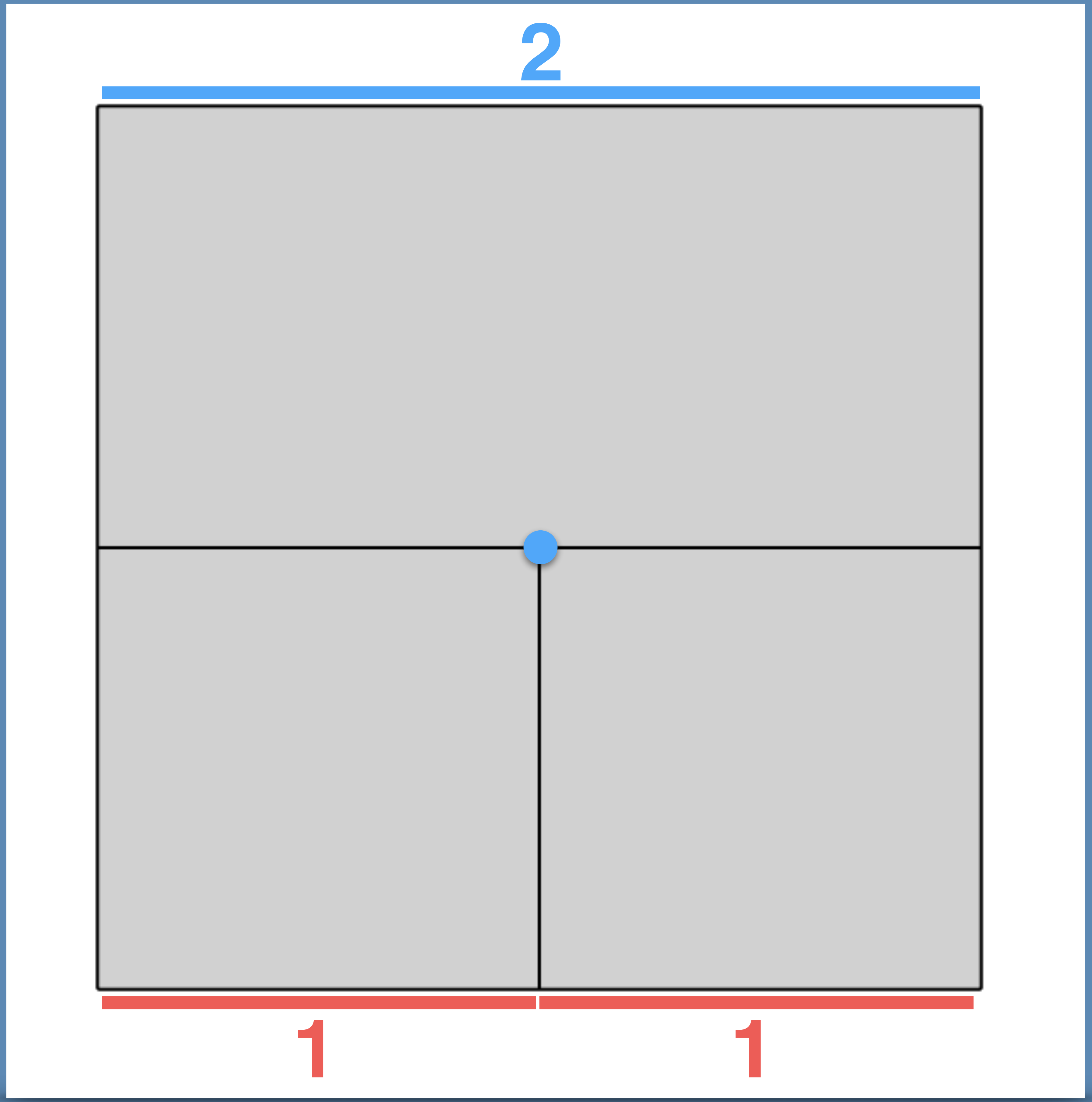
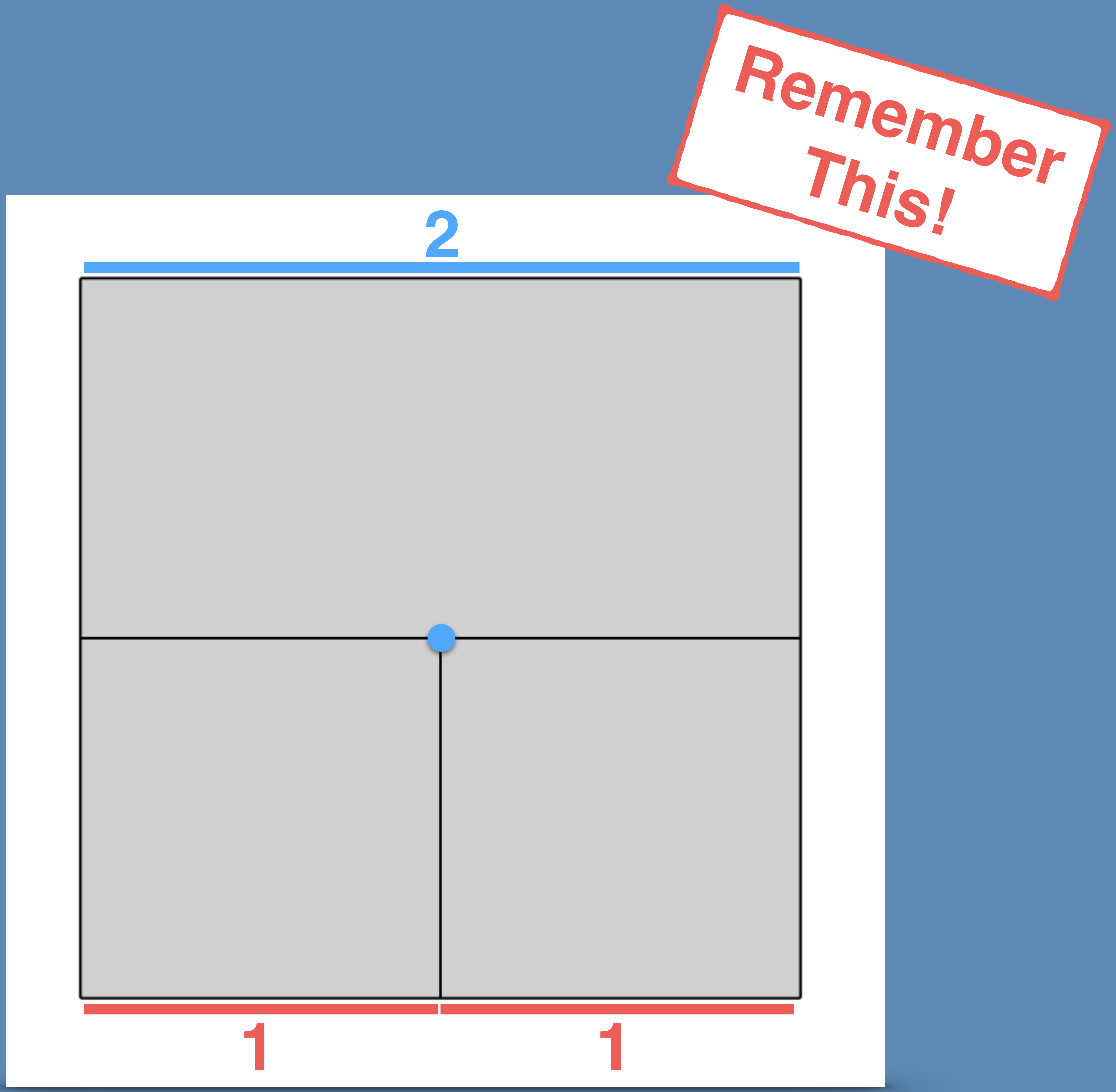



Non-uniform knot intervals

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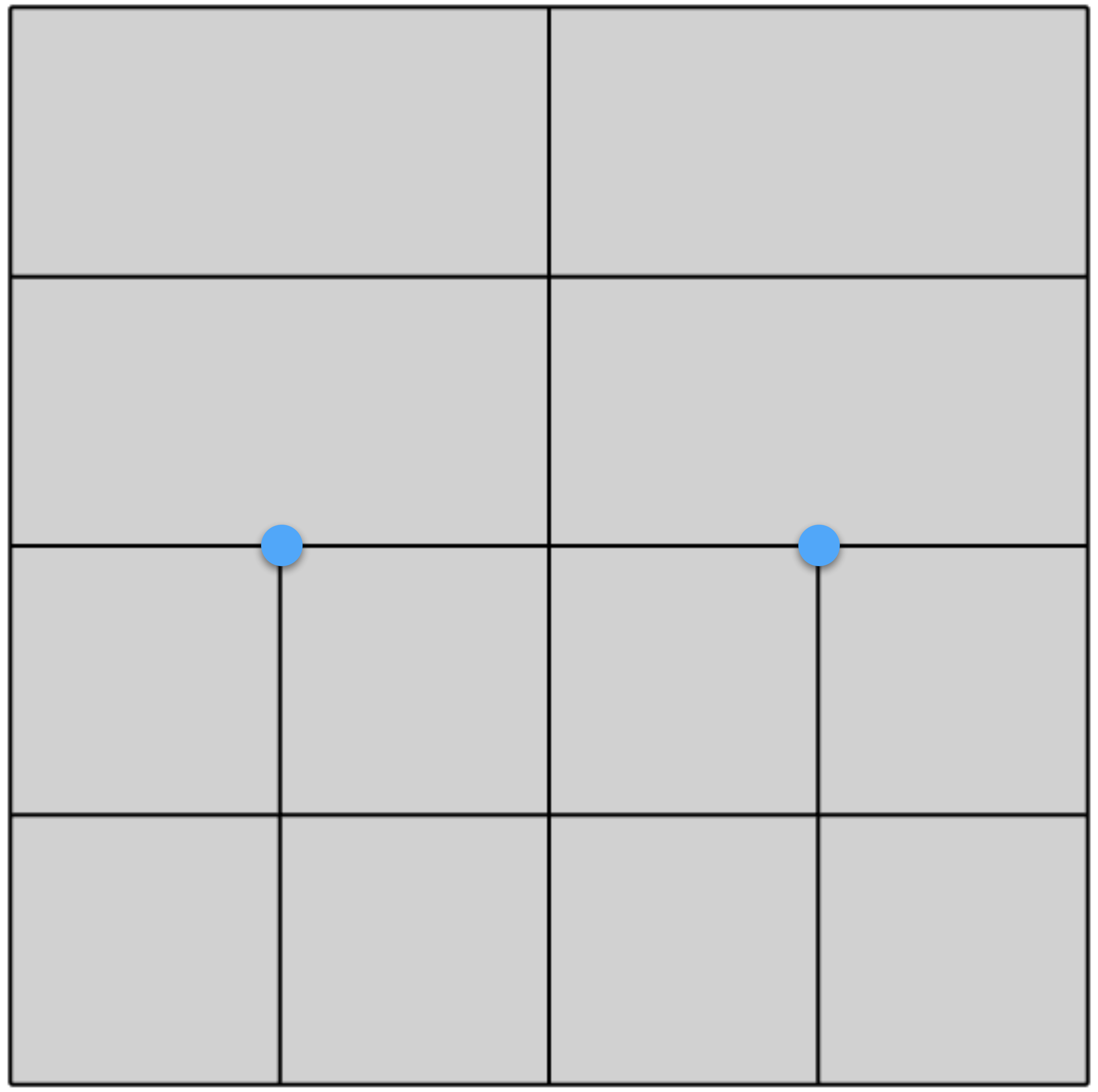
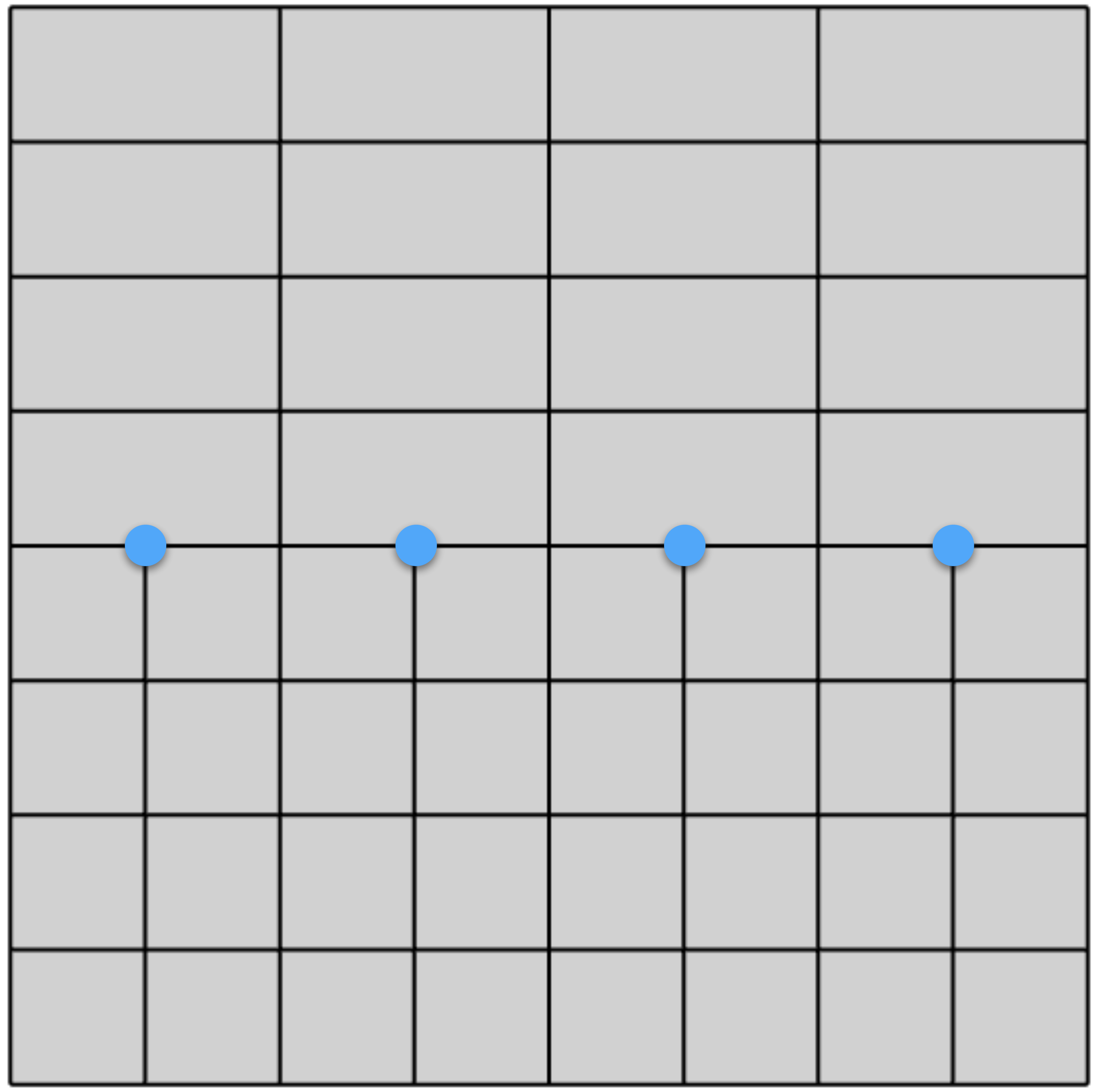
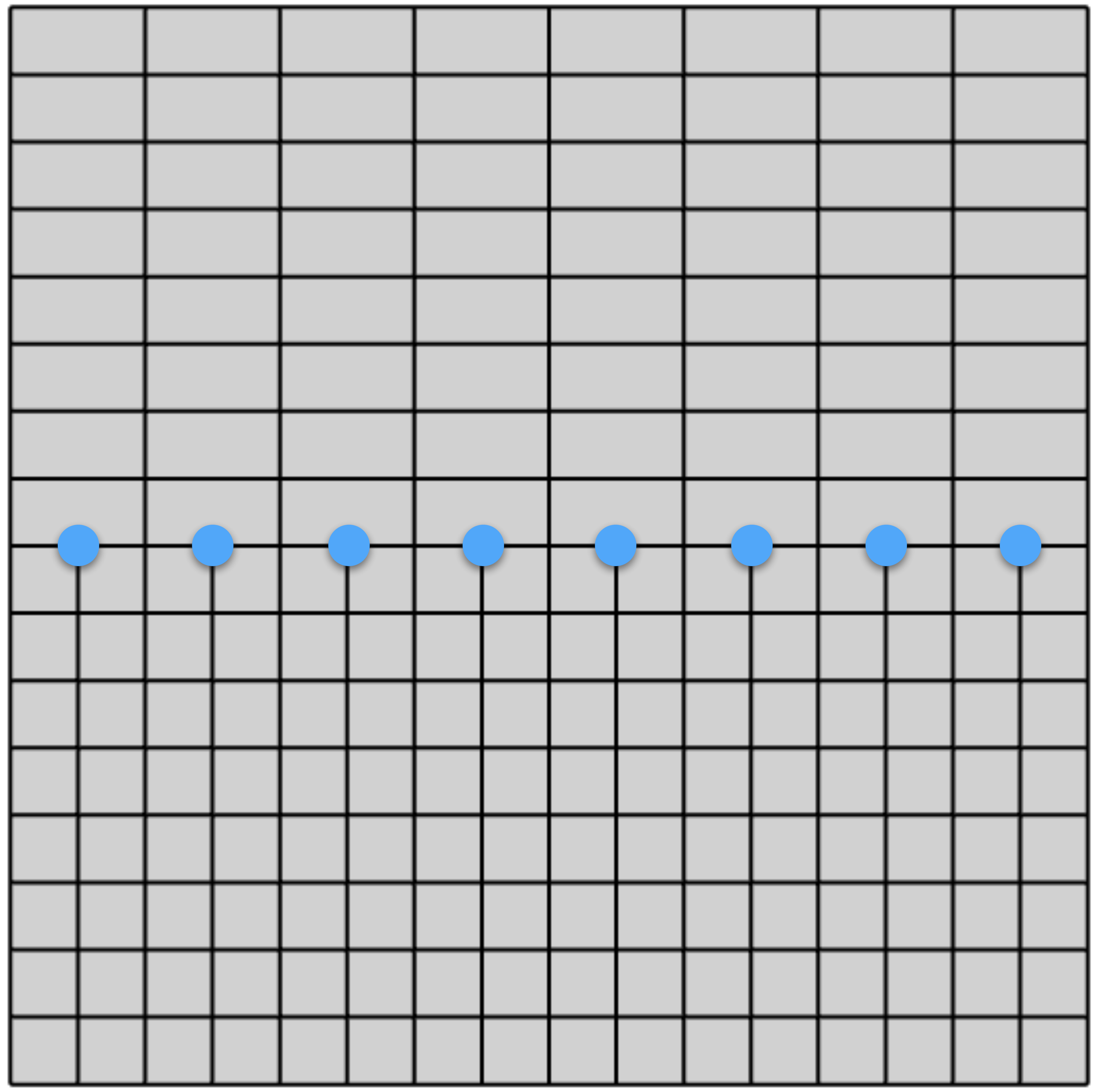
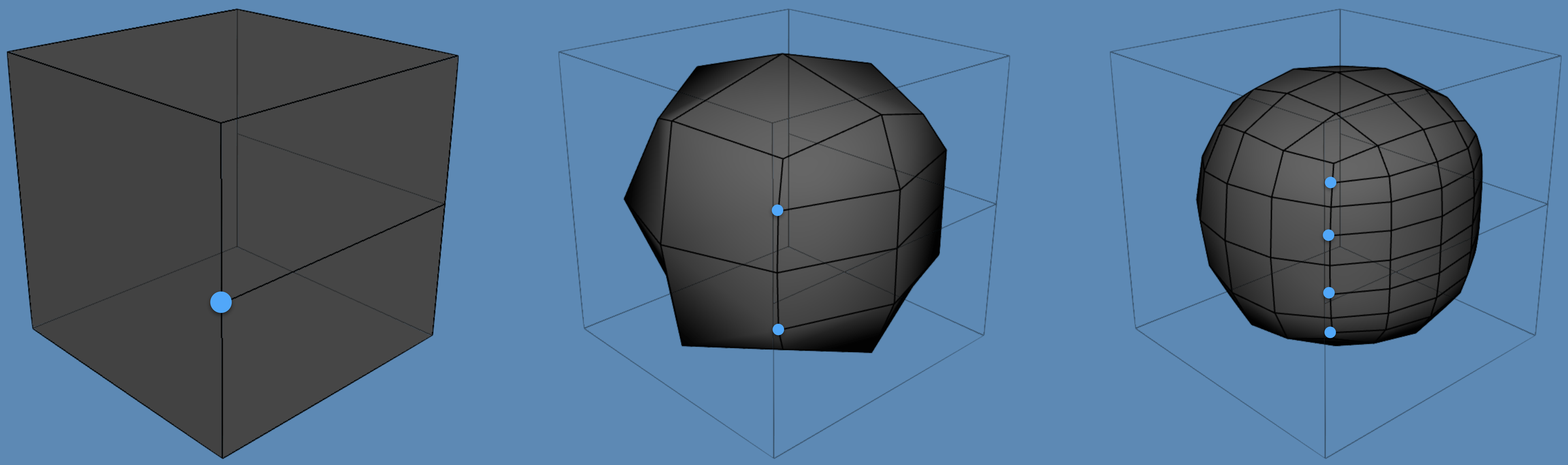




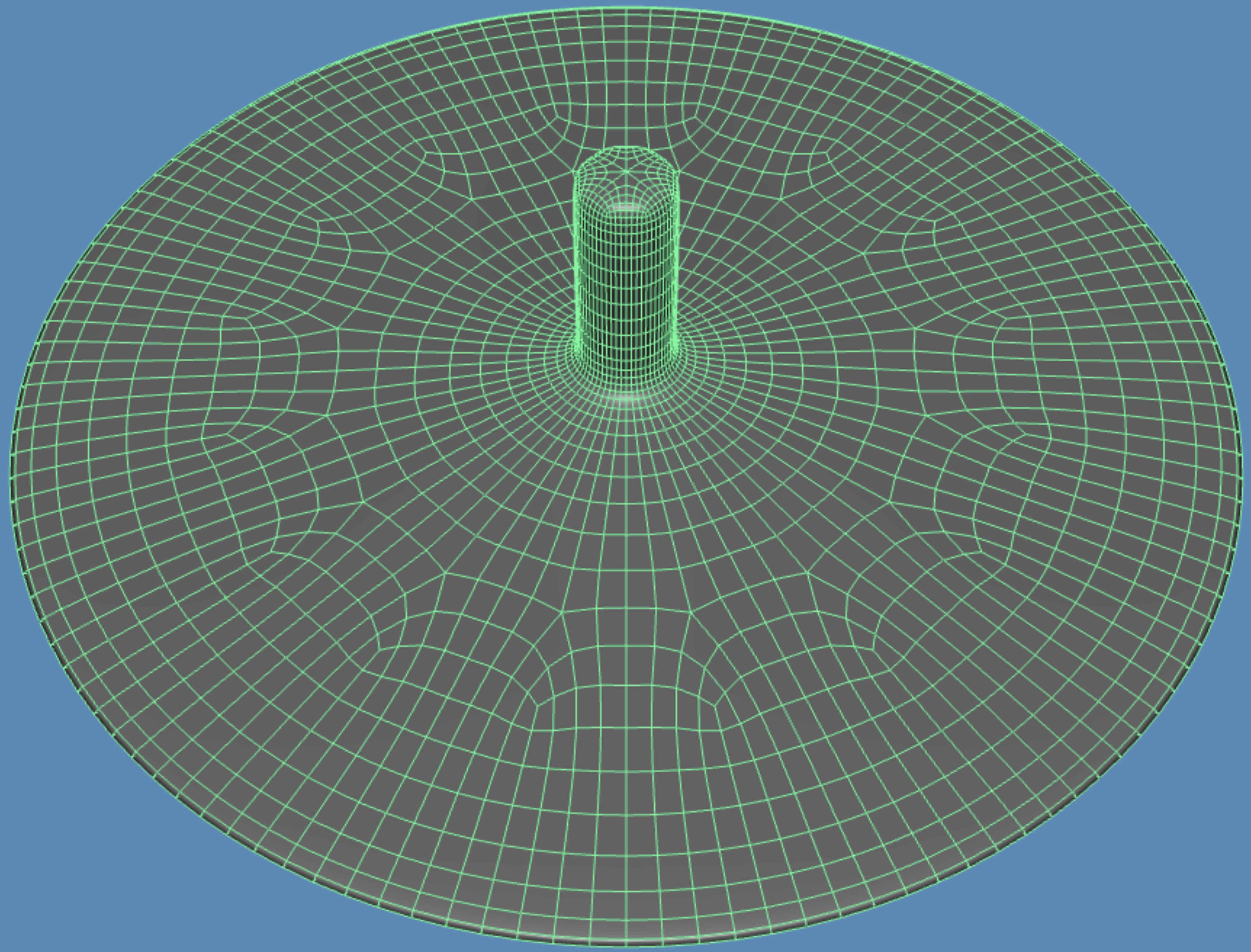


# Dyadic T-mesh Subdivision



# Spinning Top: T-mesh Subdivision

- Need for T-vertex-type topology
- And T-mesh subdivision scheme



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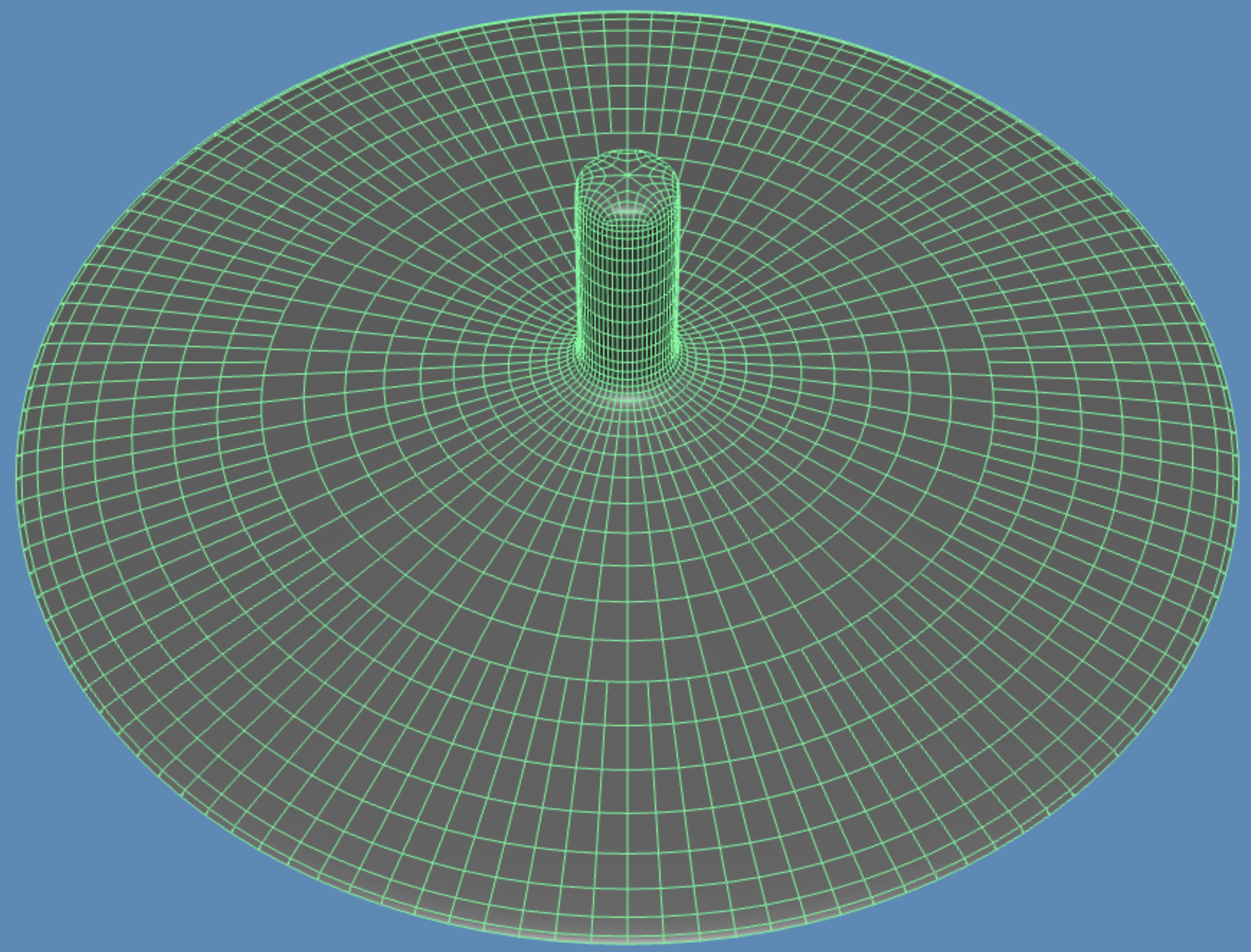

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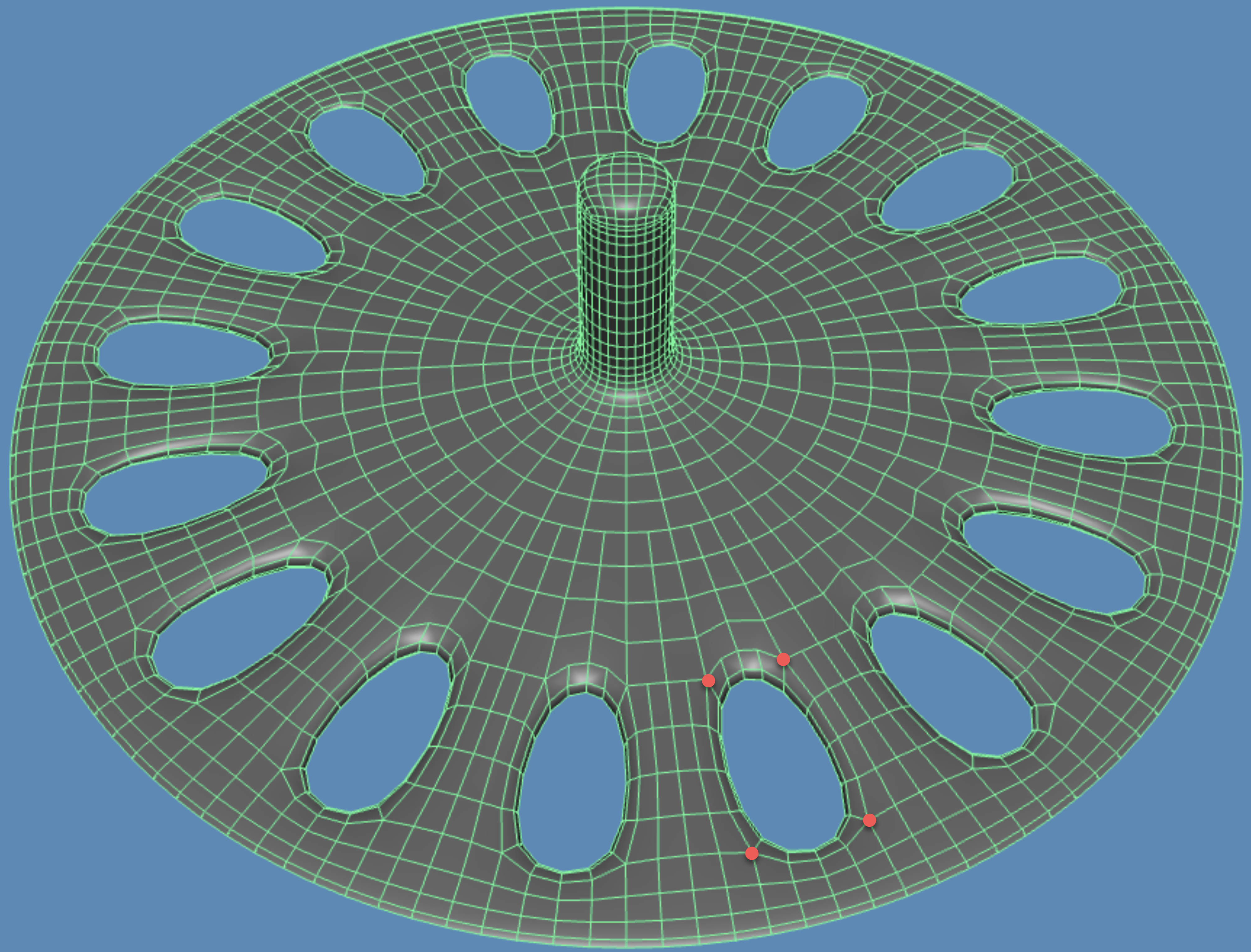

# Literature

- **Recursively generated B-spline surfaces on arbitrary topological meshes** (1978).
  CATMULL, E., AND CLARK, J.
- **Interactive multiresolution mesh editing** (1997).
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  SEDERBERG, T., ZHENG, J., SEWELL, D., AND SABIN, M
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*Tensor-Product Surfaces*

*Spline Surfaces*

Uniform B-Splines

*Subdivision Surfaces*

Catmull-Clark Subdivision

*Tensor-Product Surfaces*

*Spline Surfaces*

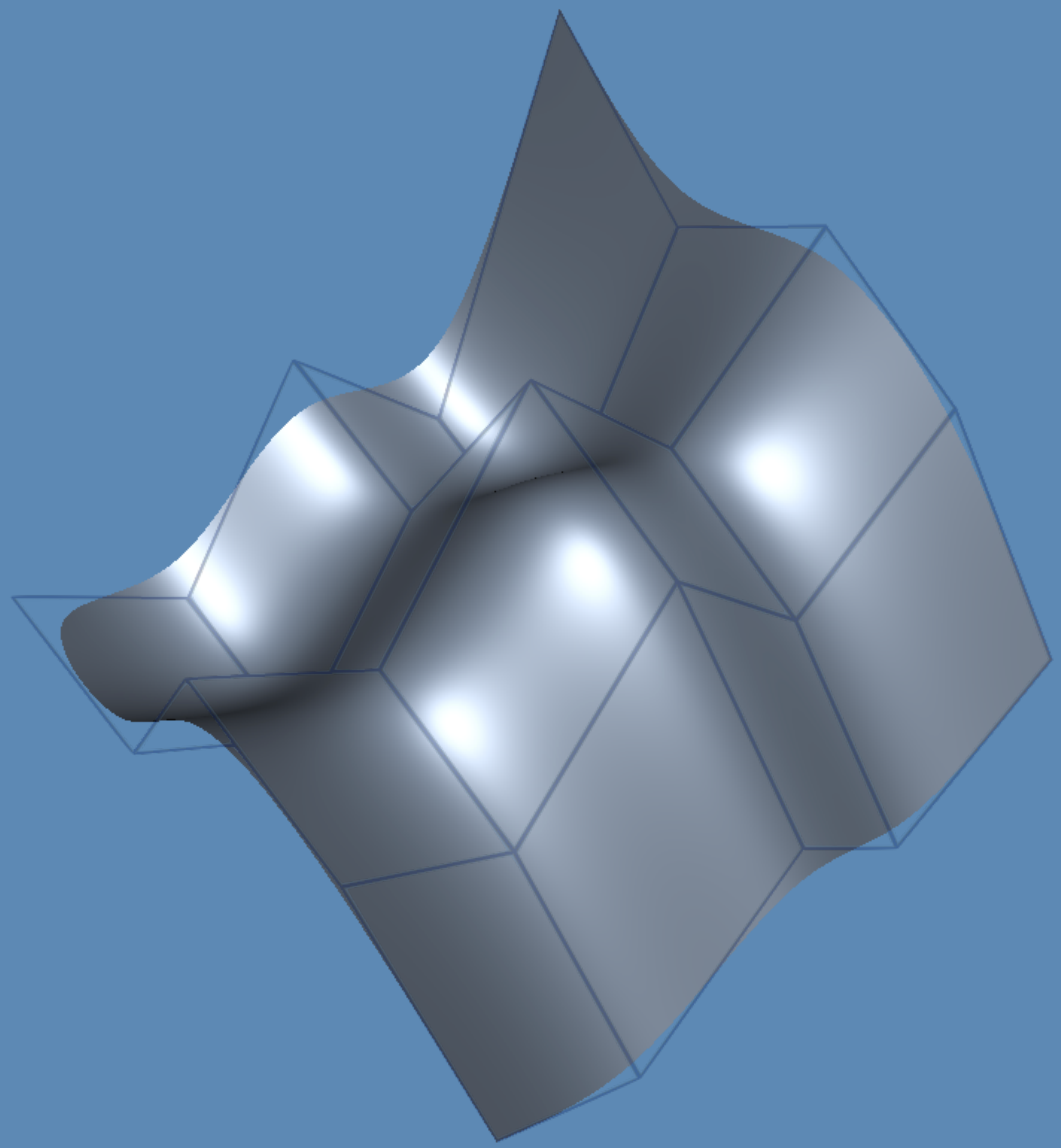
Uniform B-Splines

Knot Doubling / Factorization

*Subdivision Surfaces*

Catmull-Clark Subdivision

# B-Spline Surfaces: Knot Doubling



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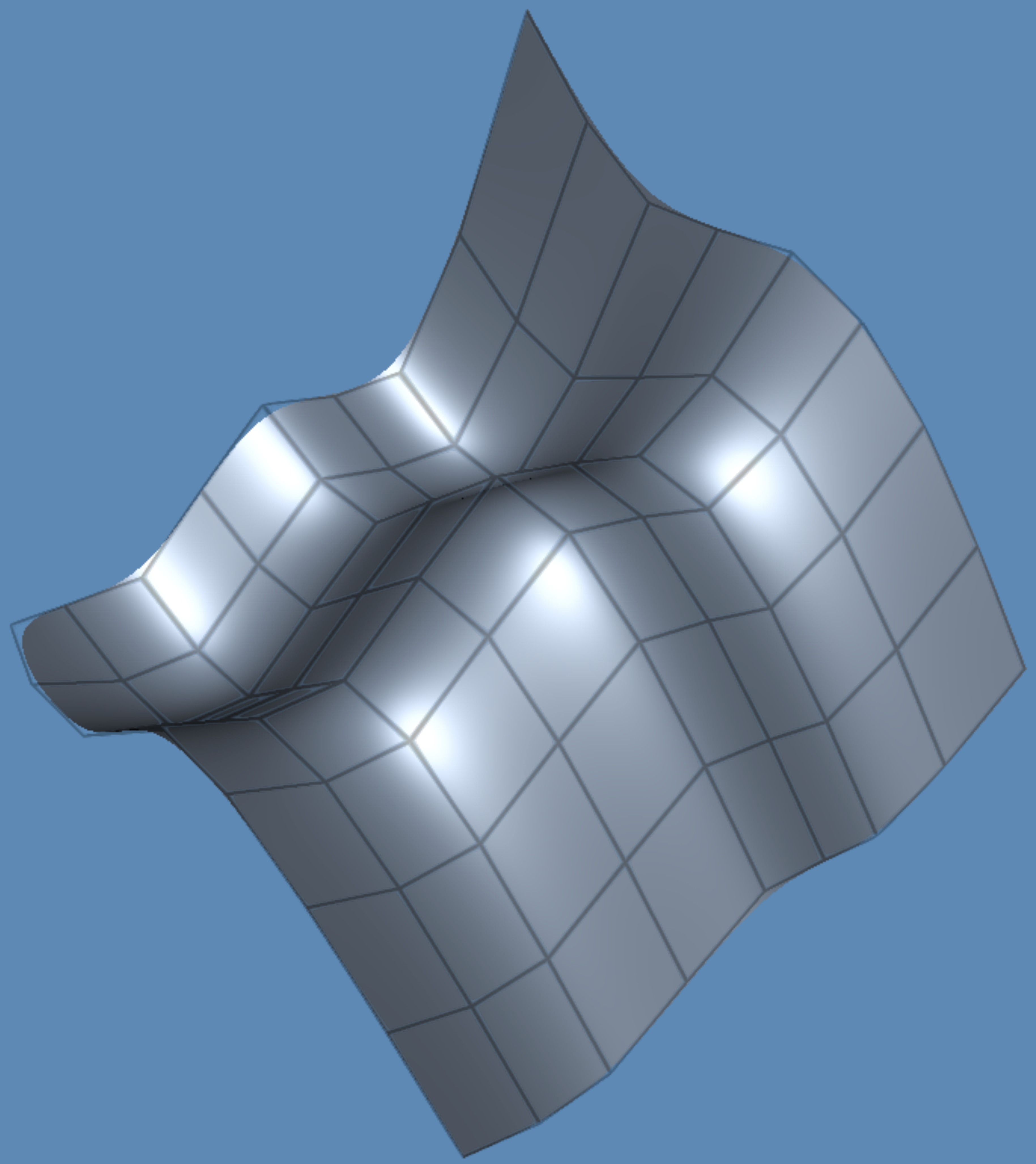

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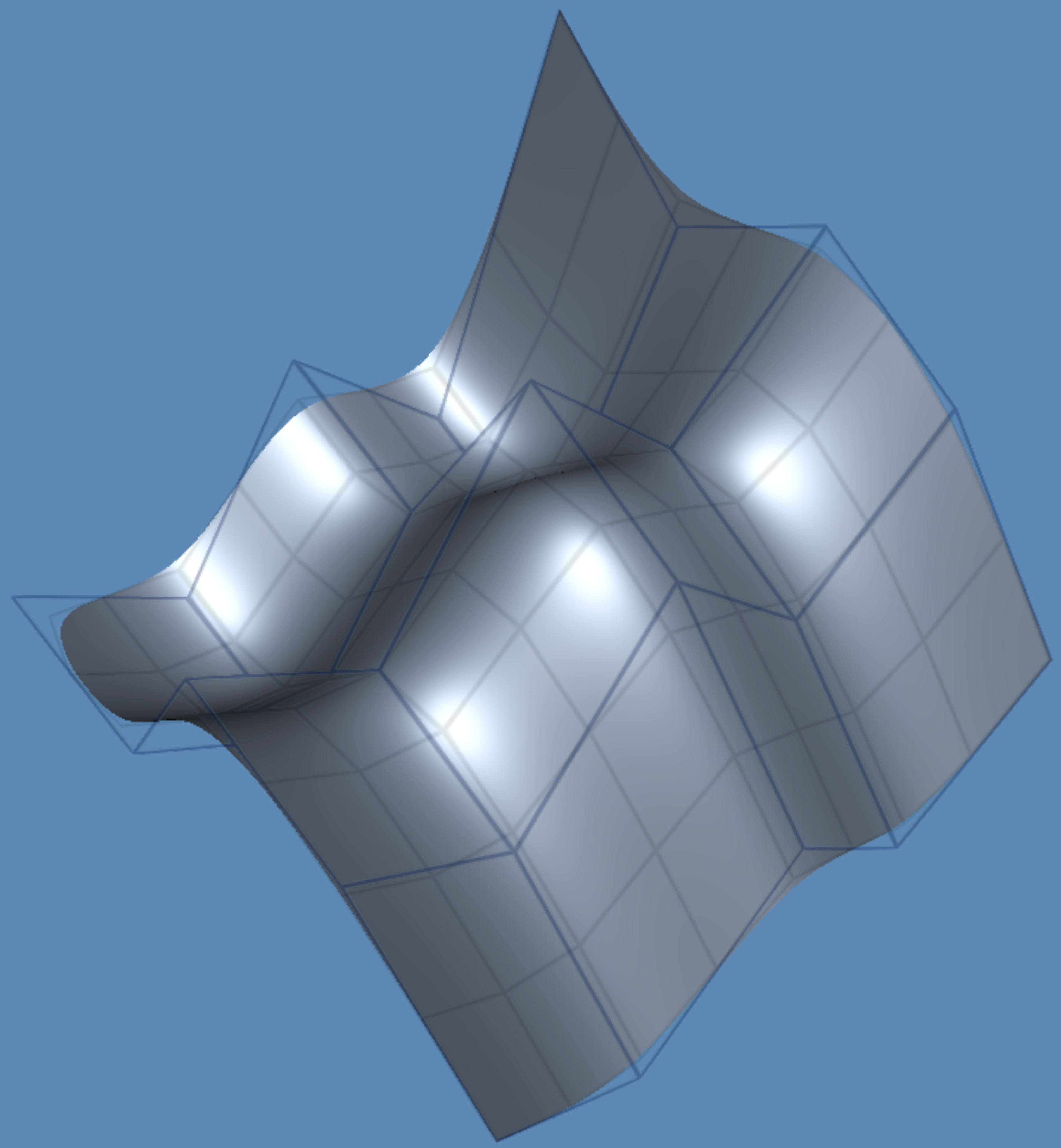

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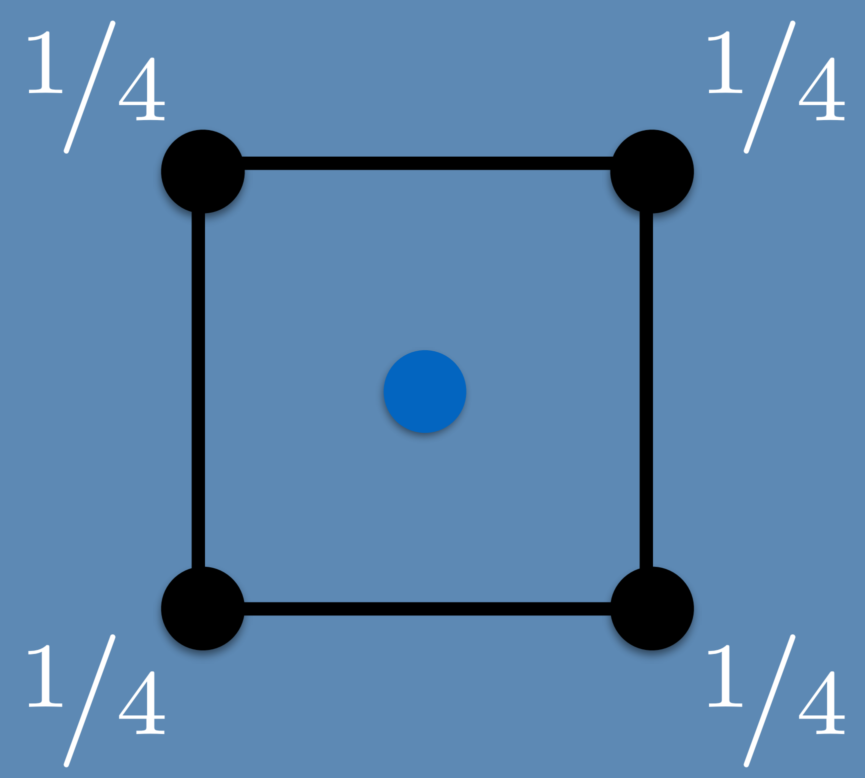


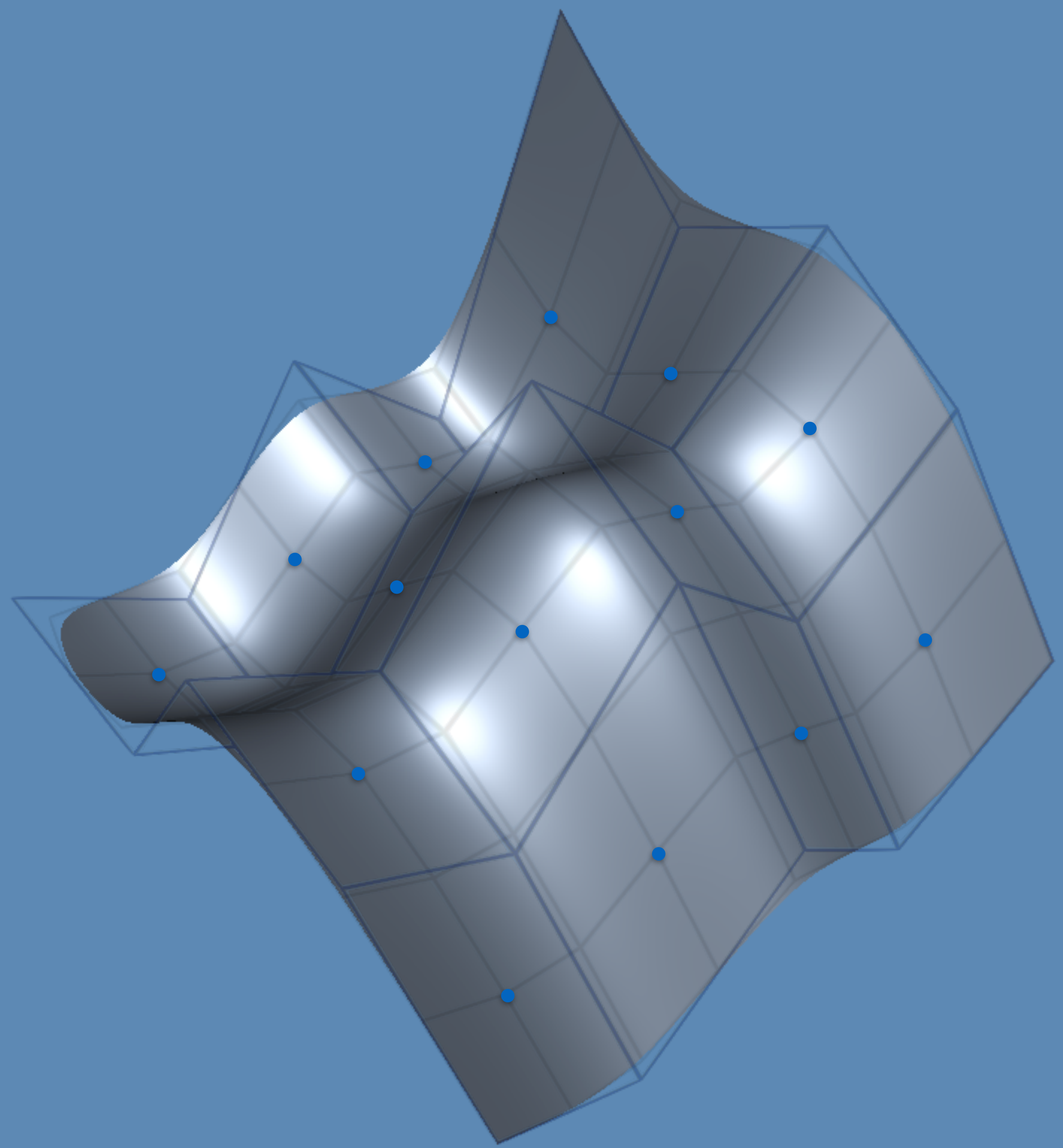

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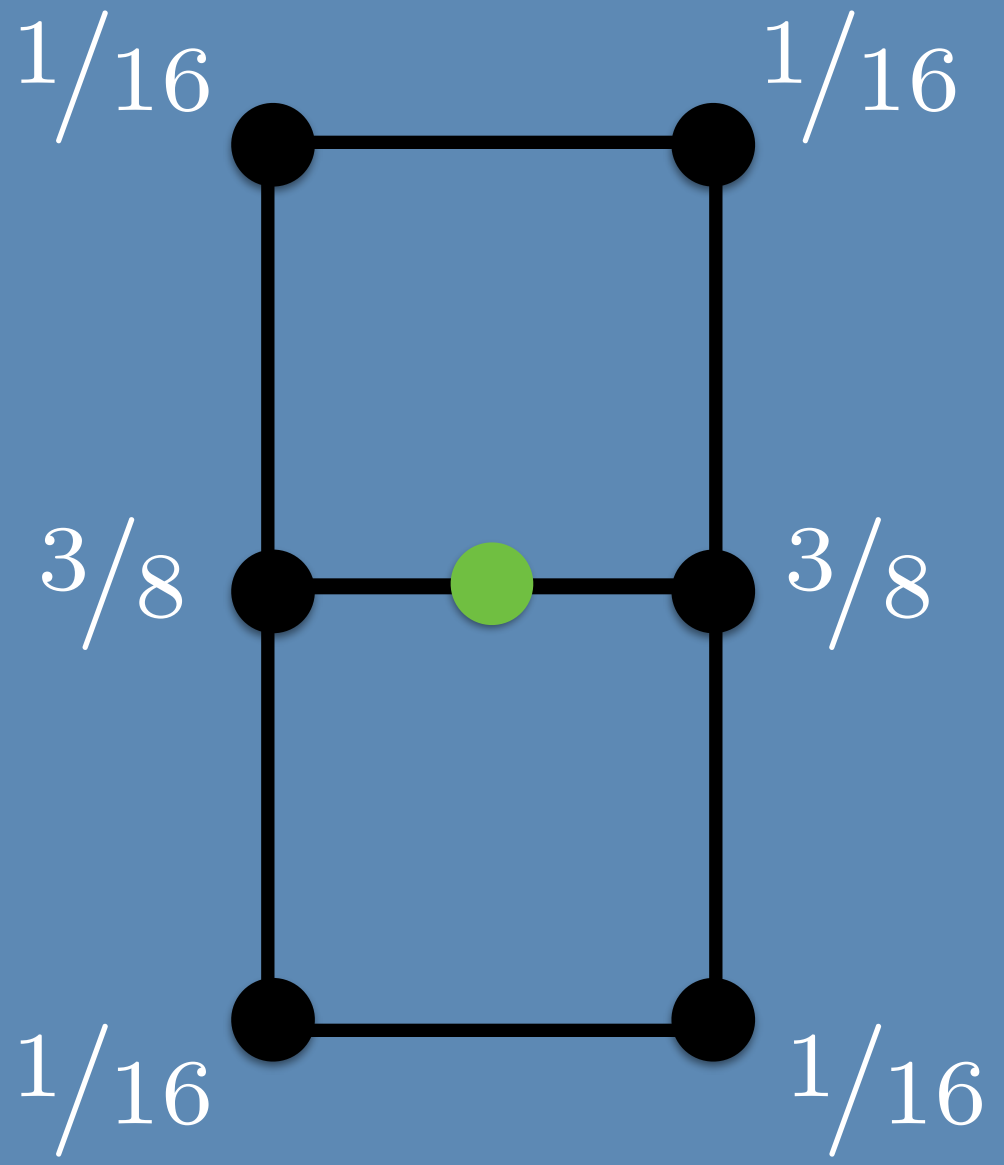


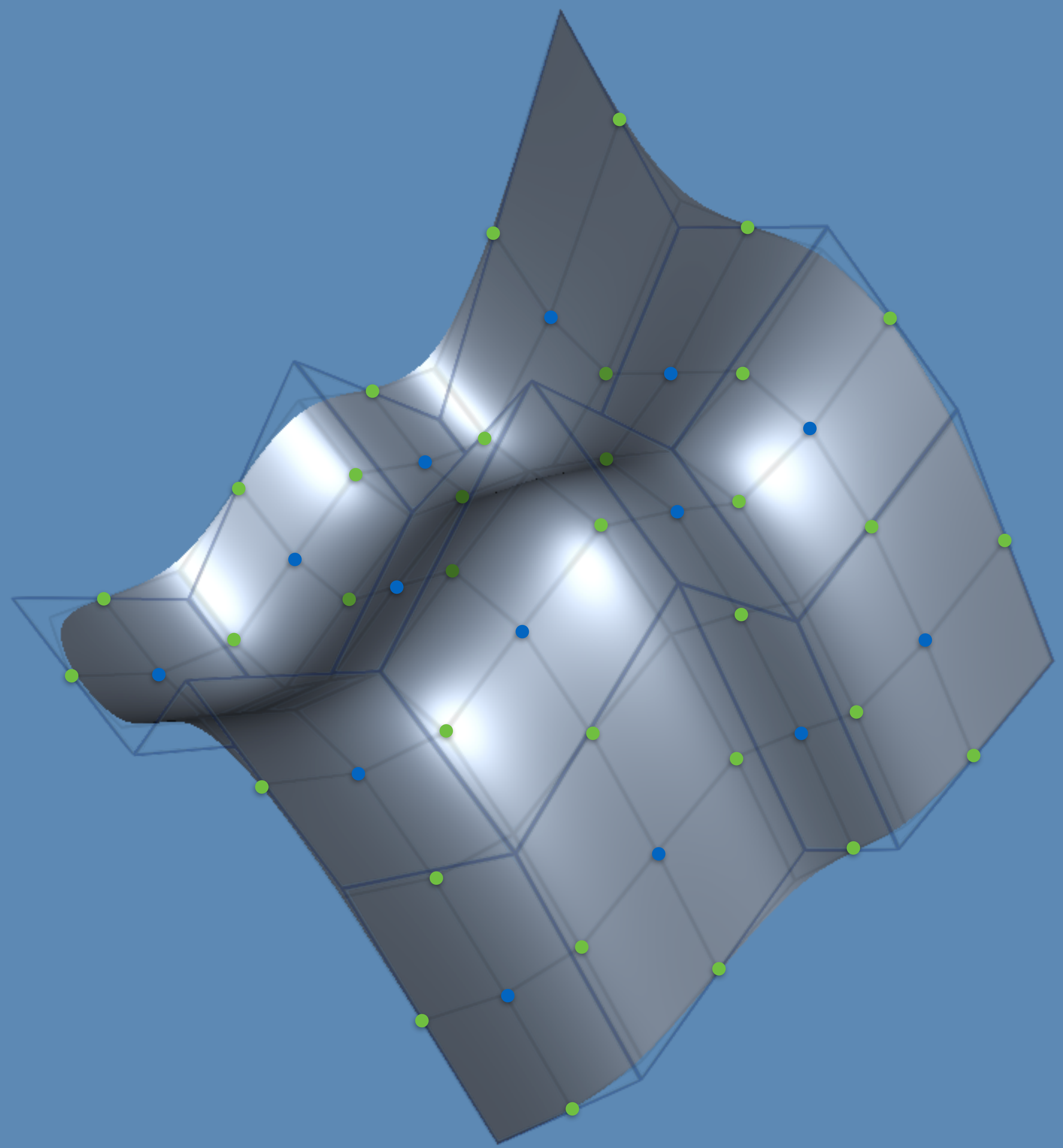

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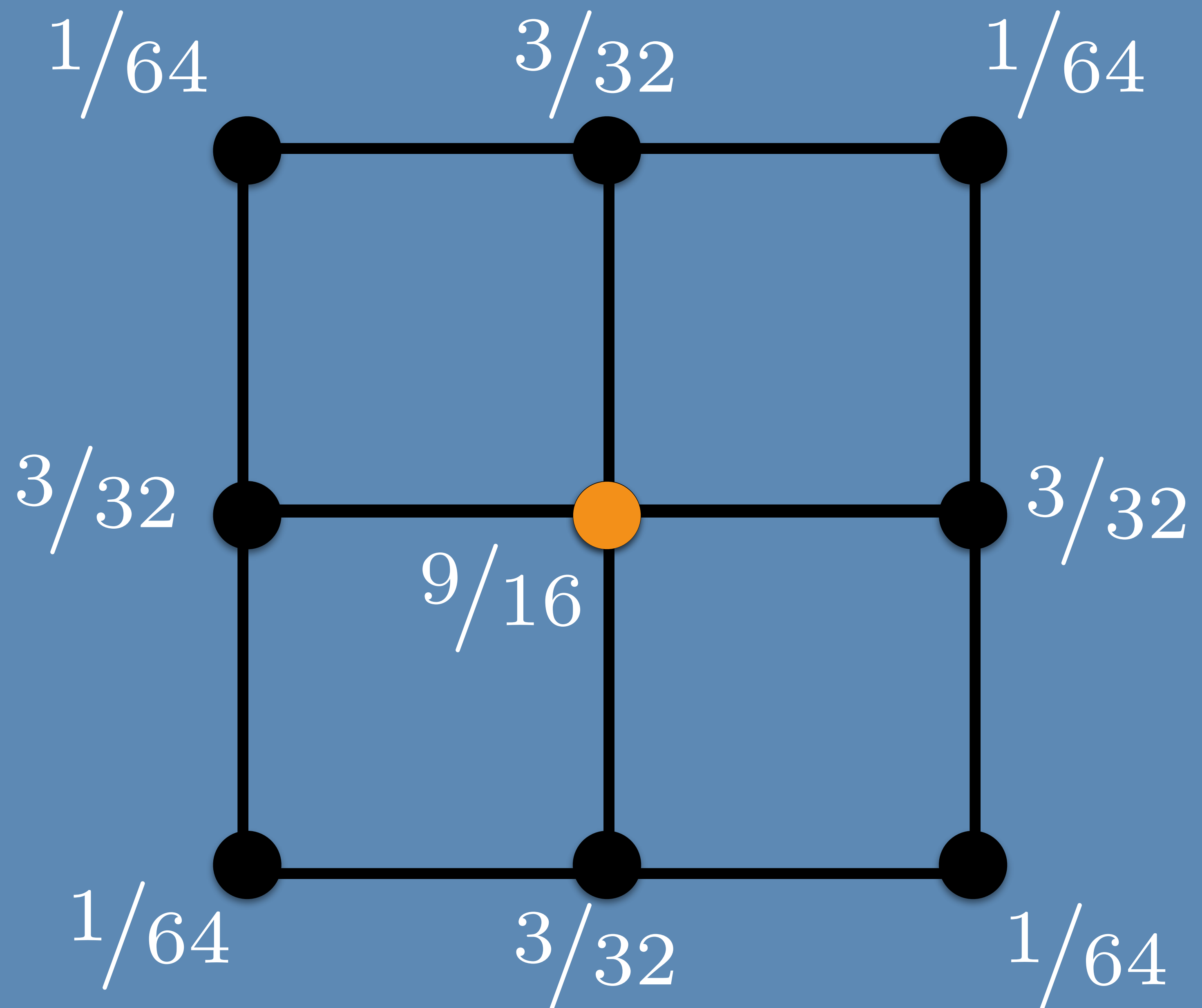
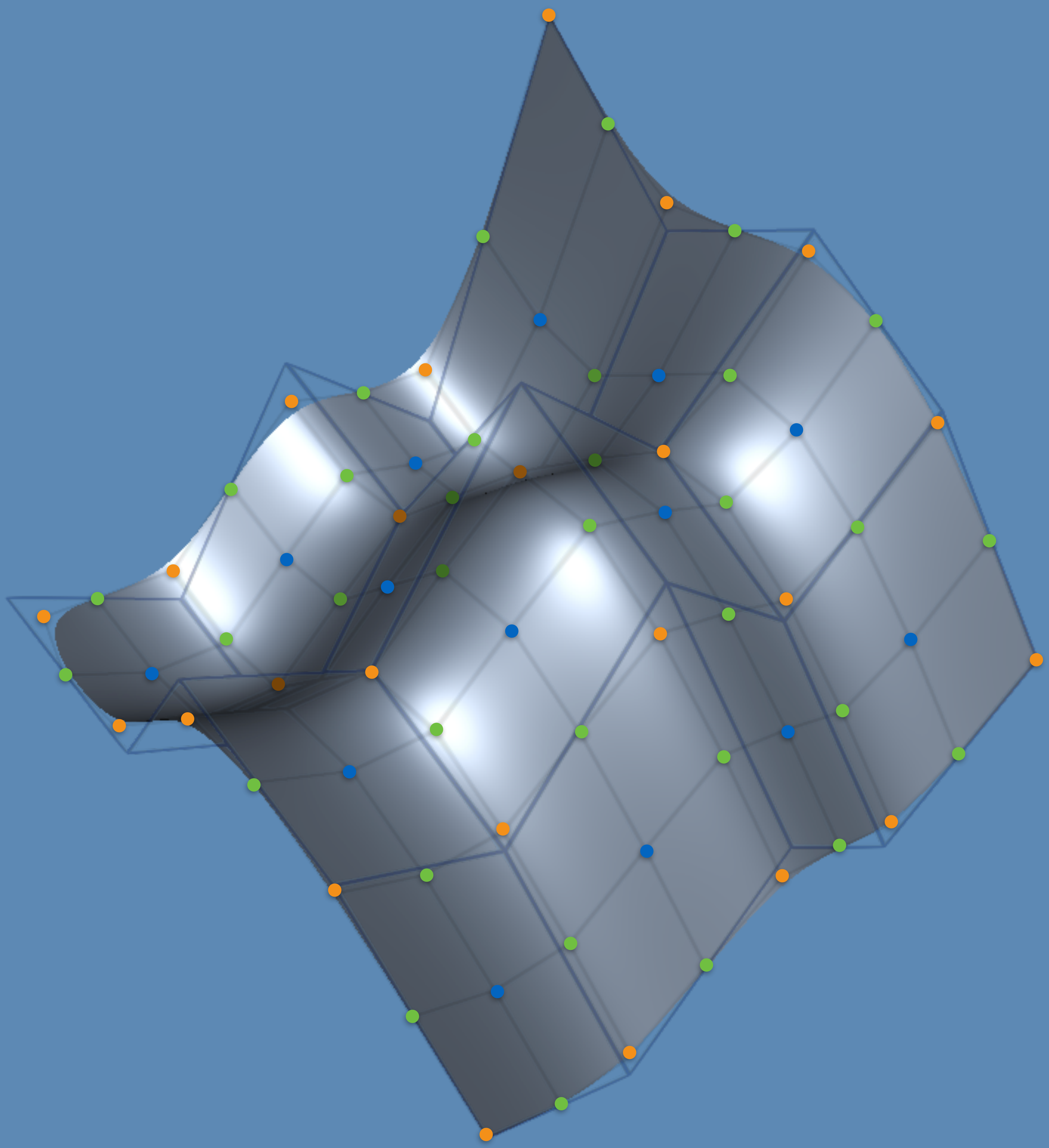
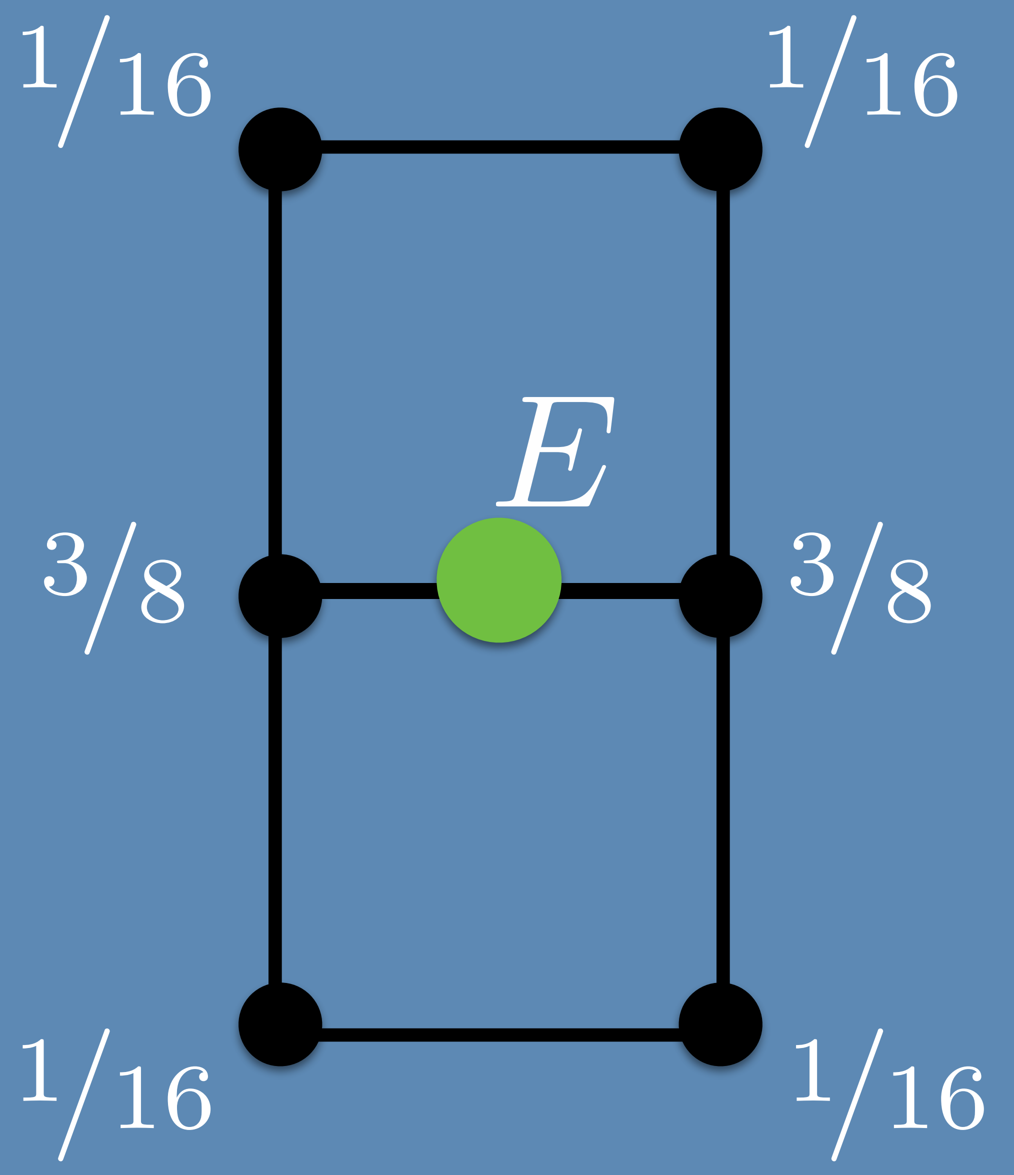


# Catmull-Clark Factorization



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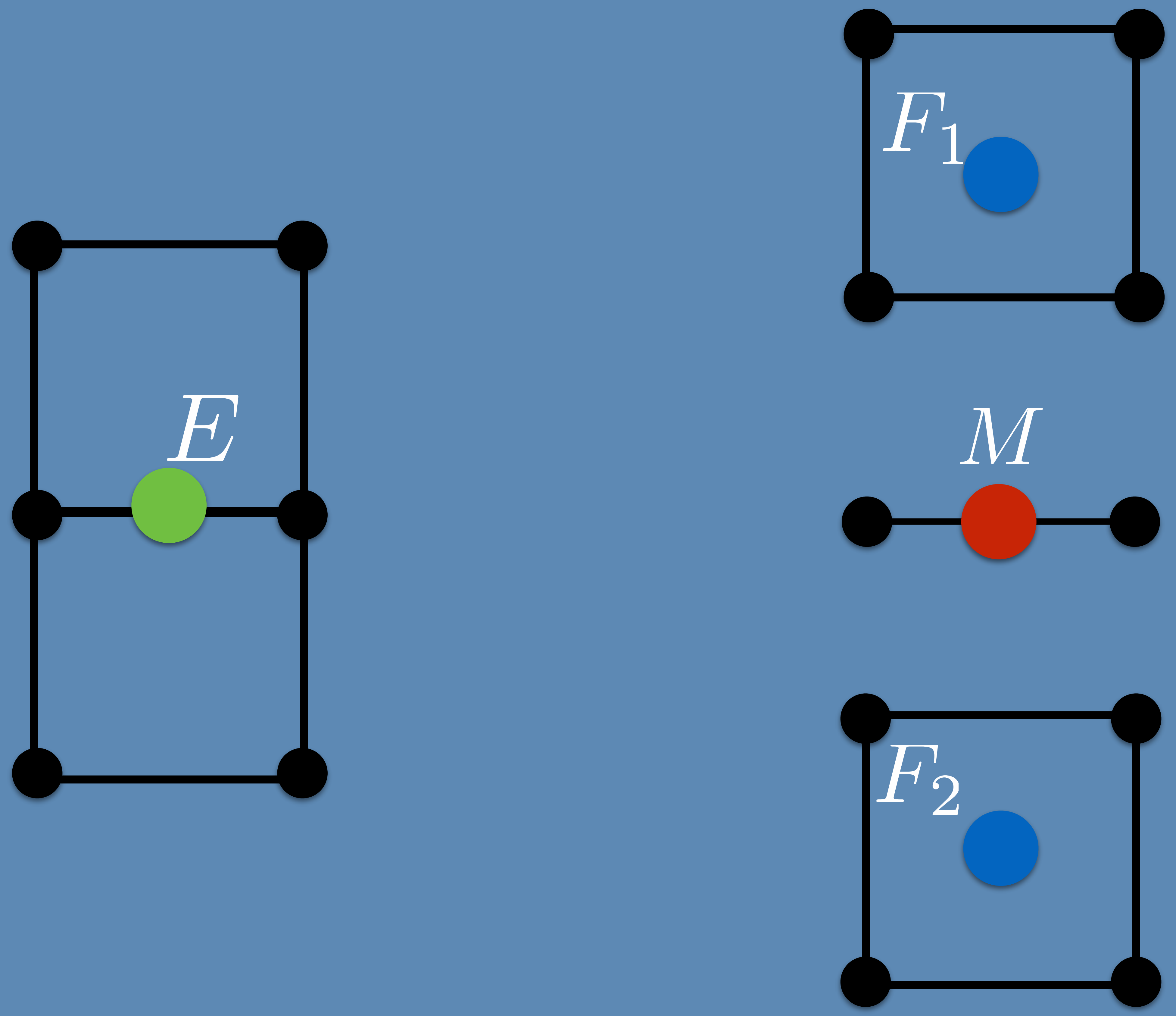

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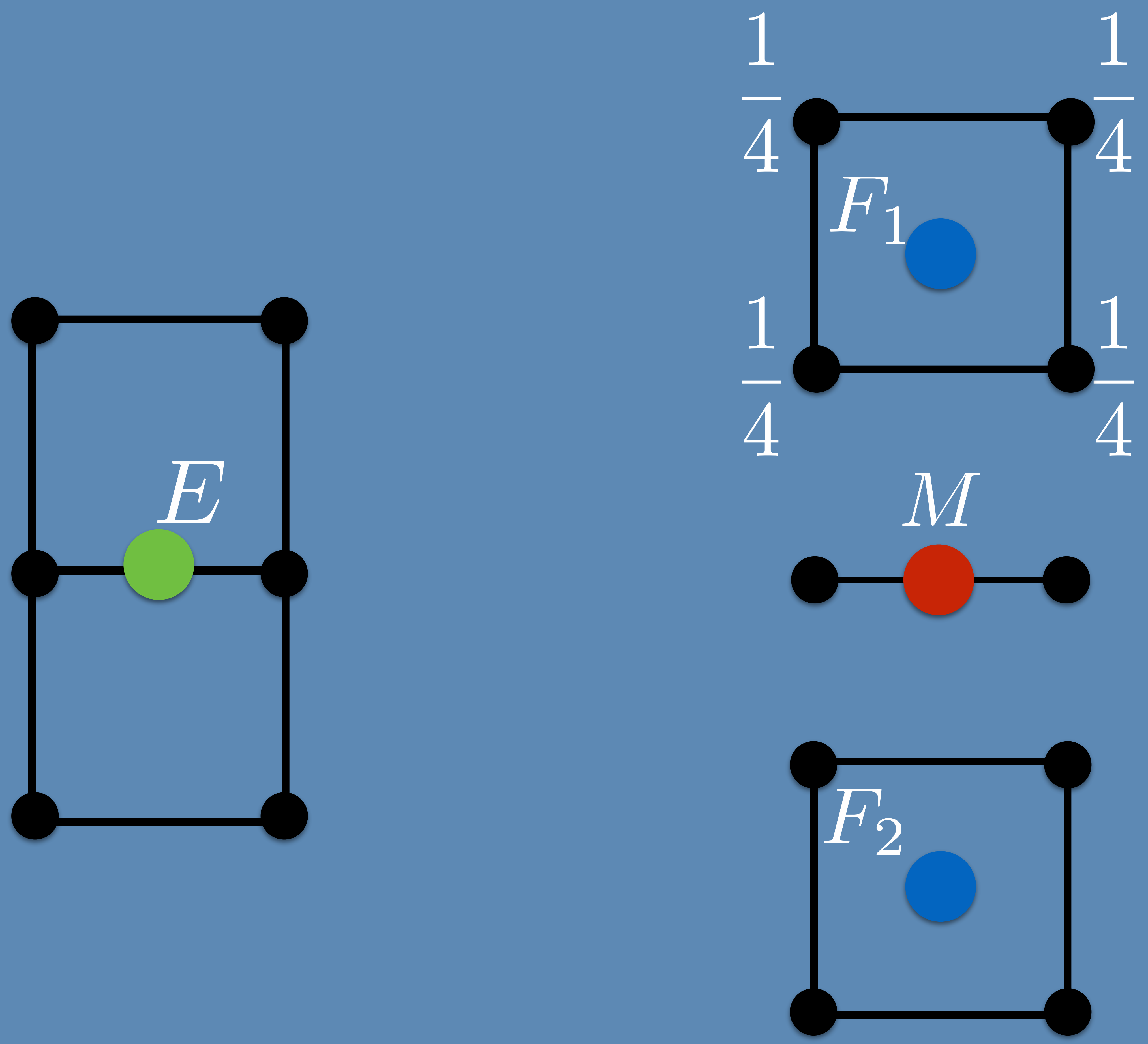

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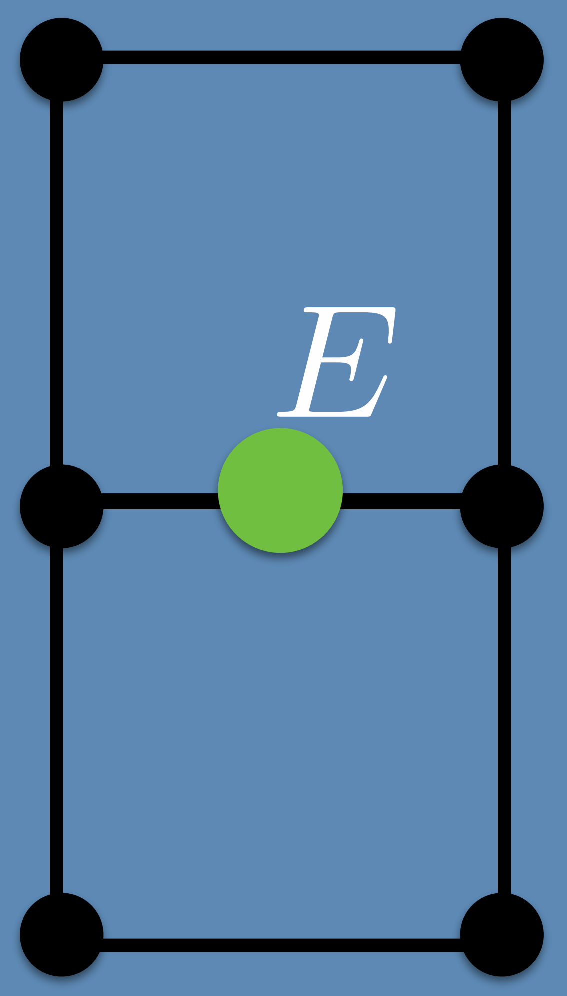


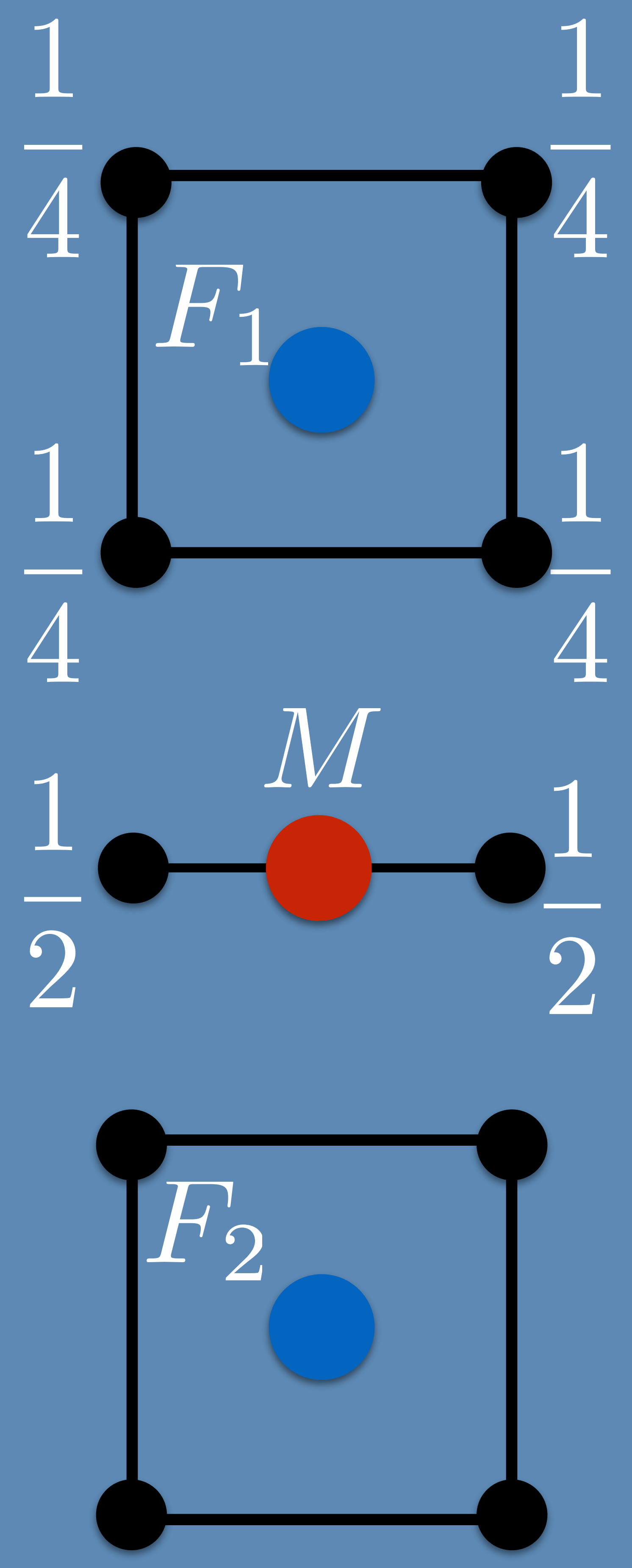

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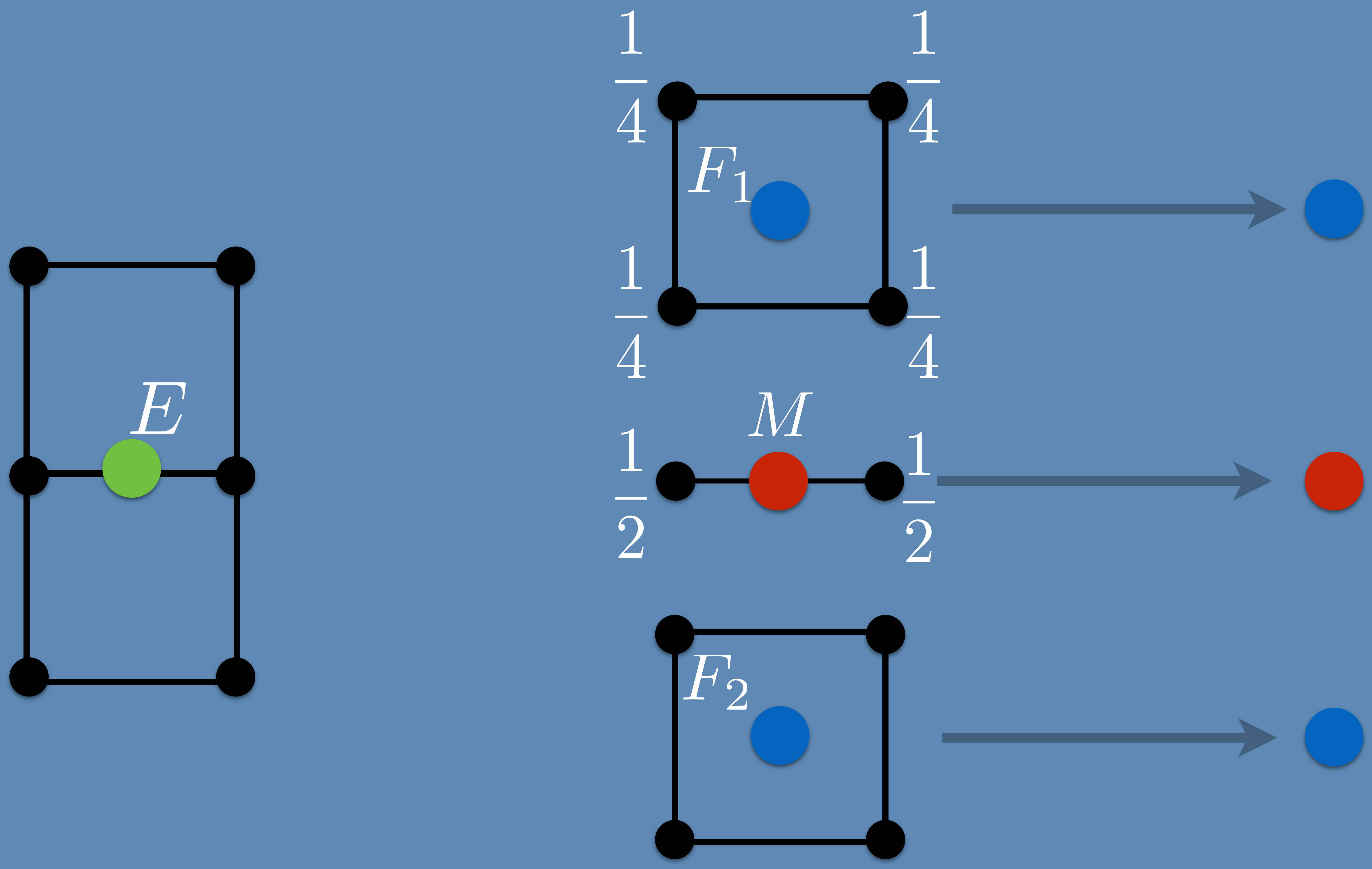

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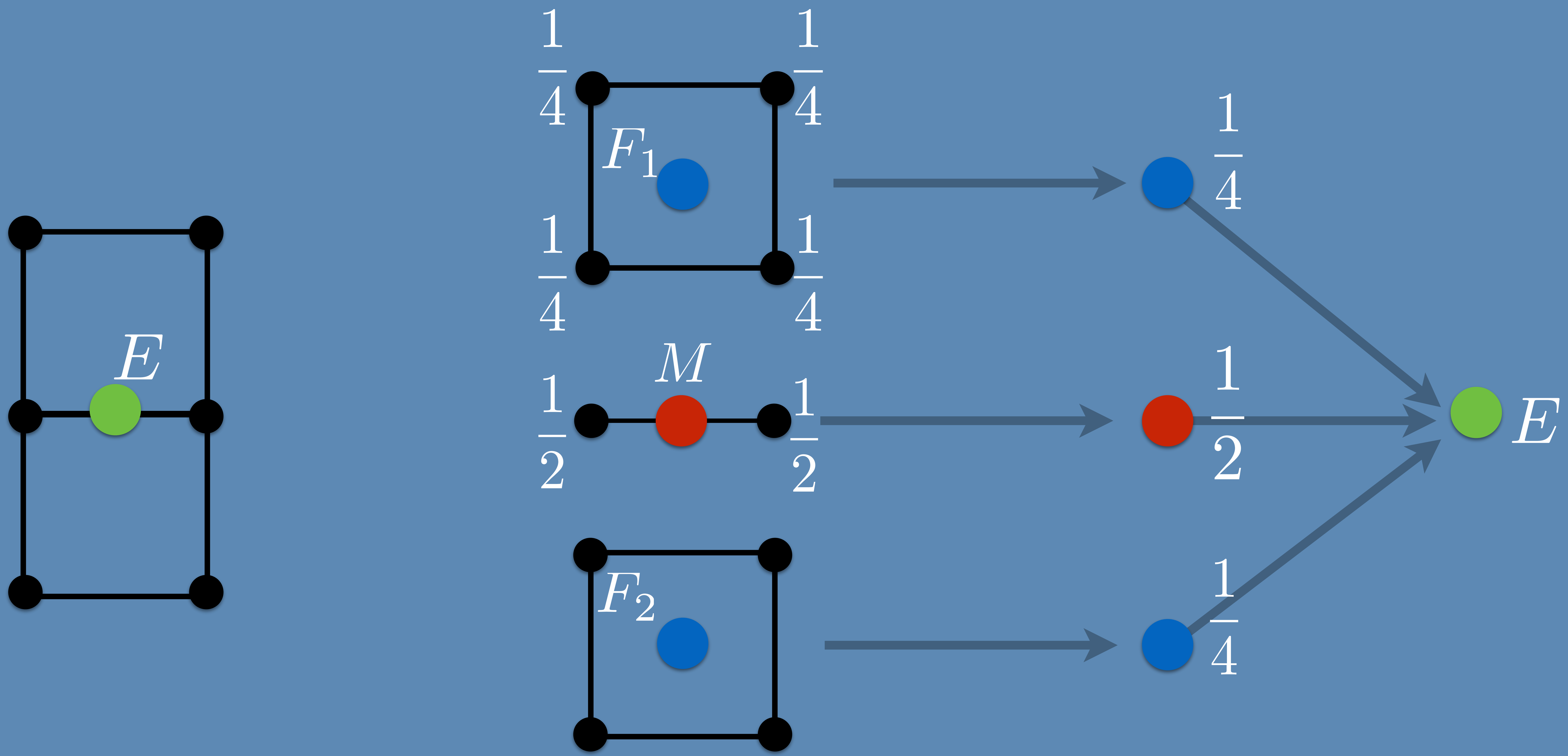
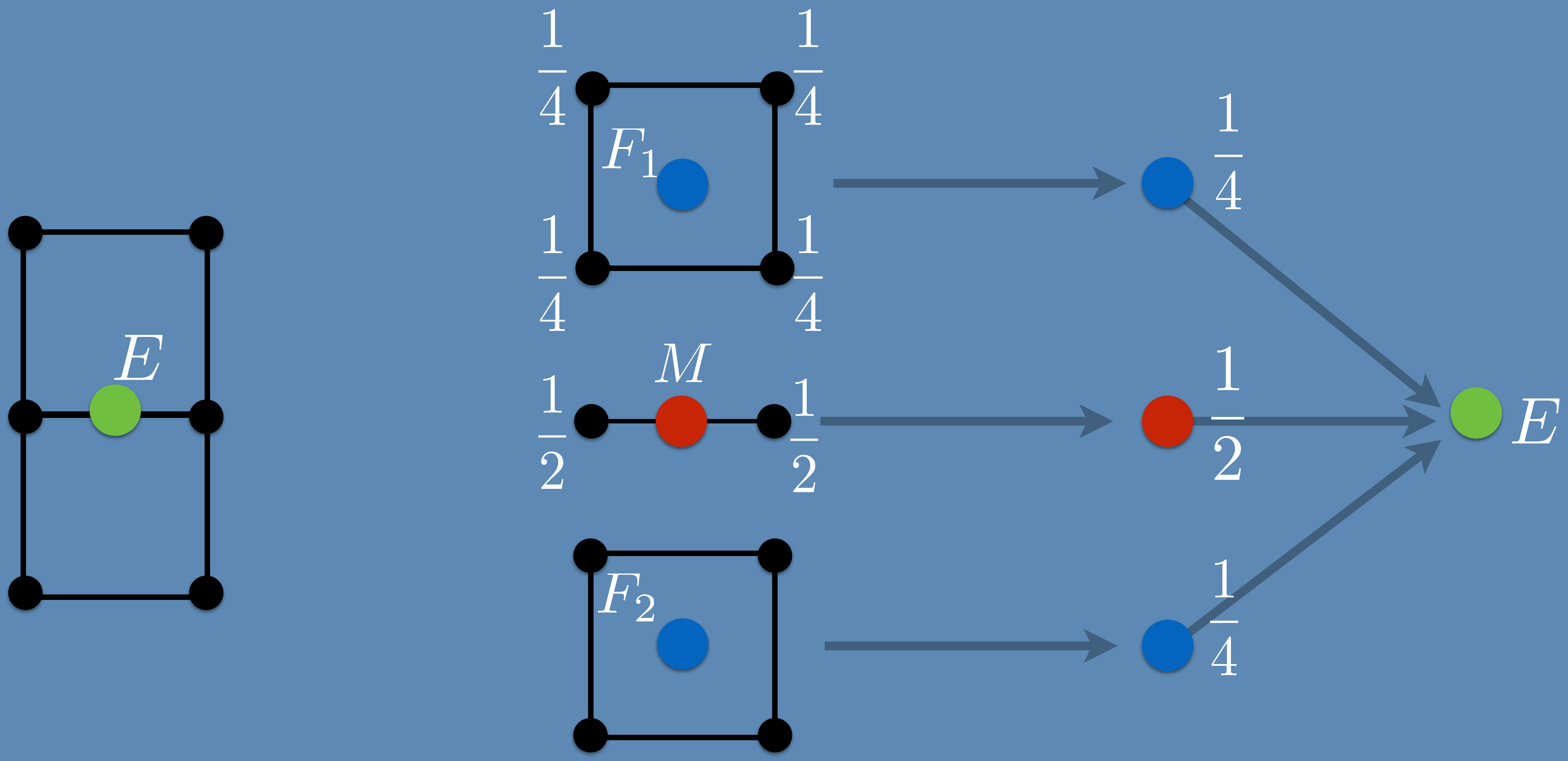


# Catmull-Clark Factorization

generalizes to arbitrary topologies



*Tensor-Product Surfaces*

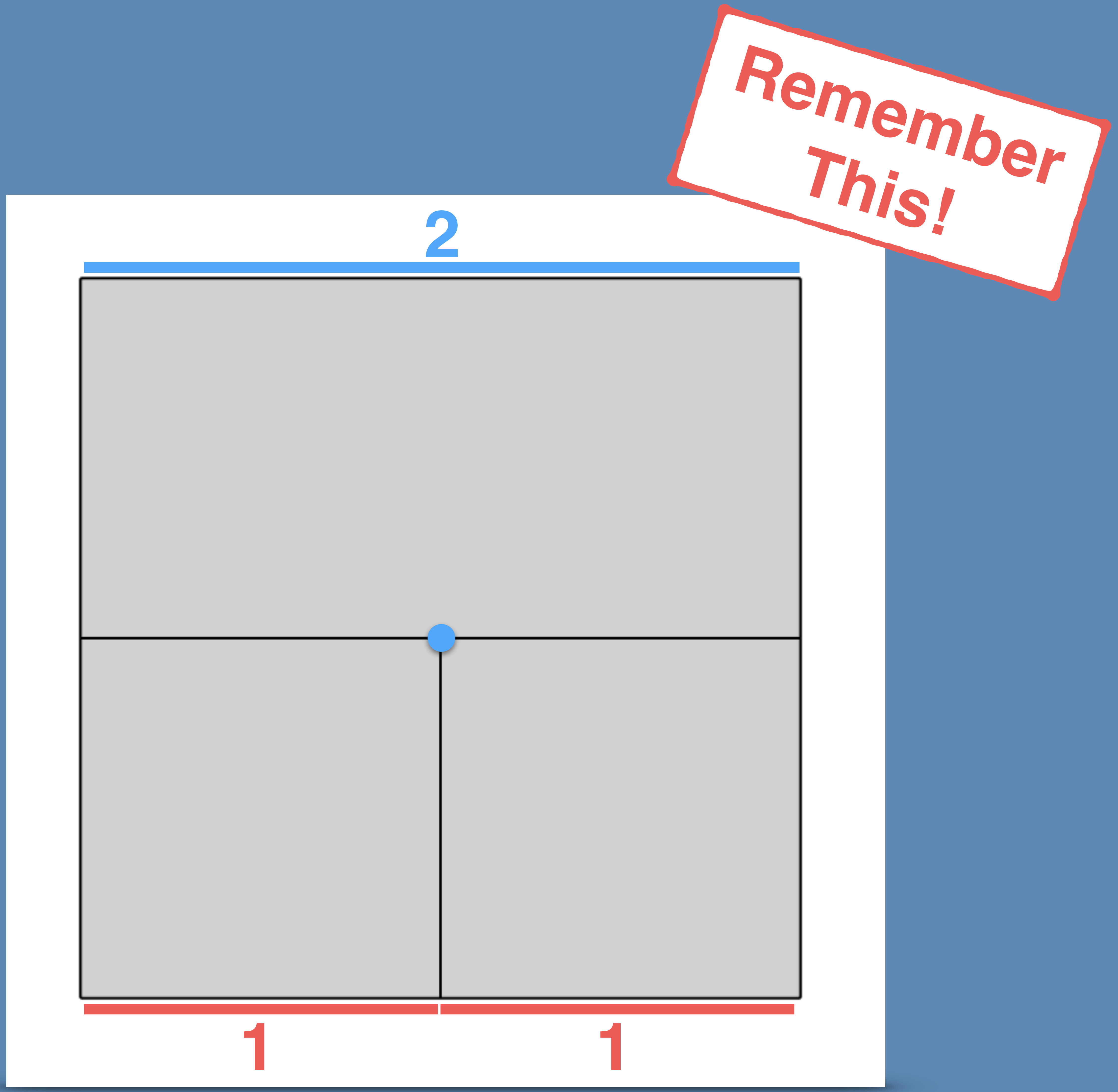
*Spline Surfaces*

Uniform B-Splines

Knot Doubling / Factorization

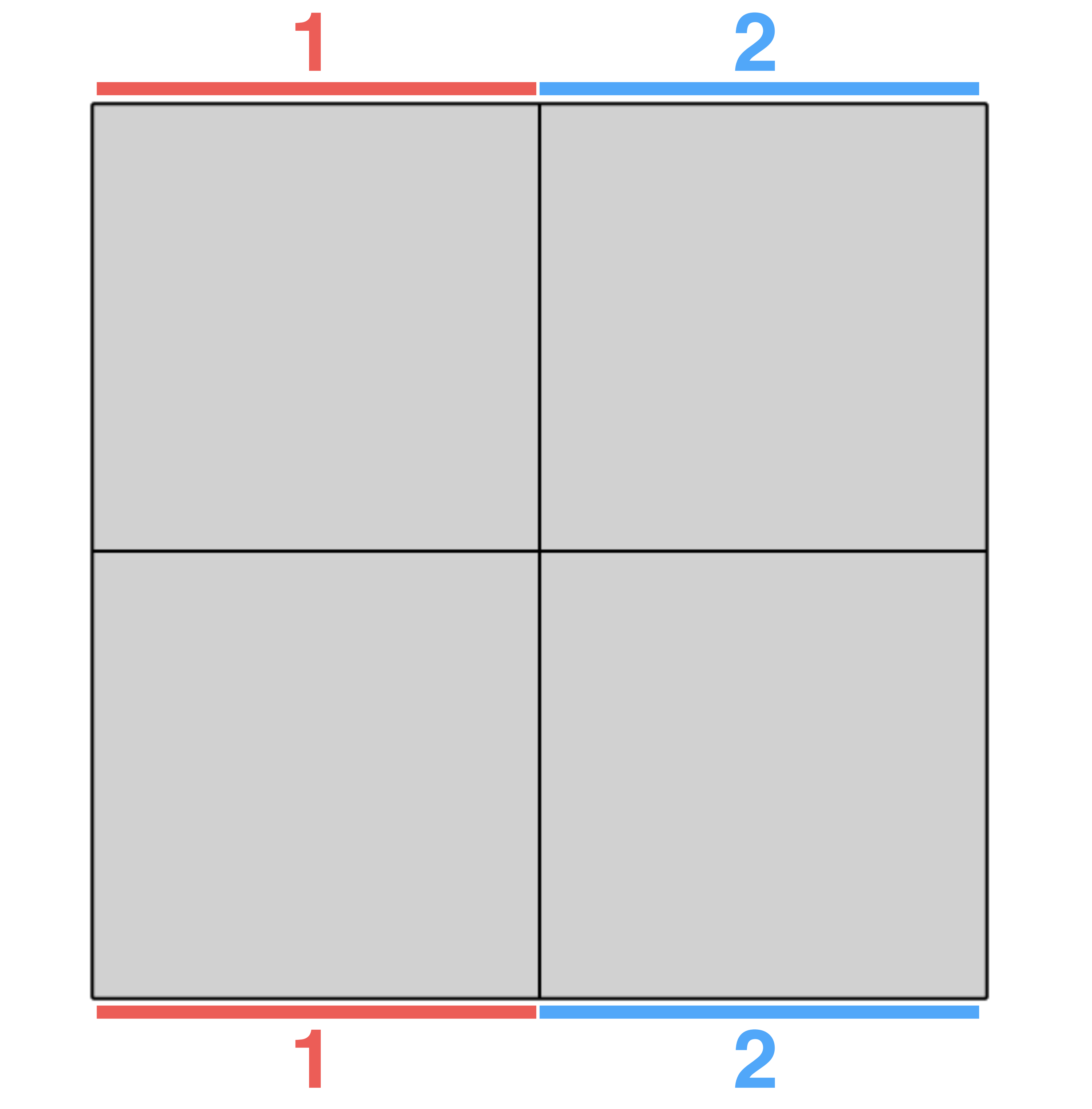
*Subdivision Surfaces*

Catmull-Clark Subdivision



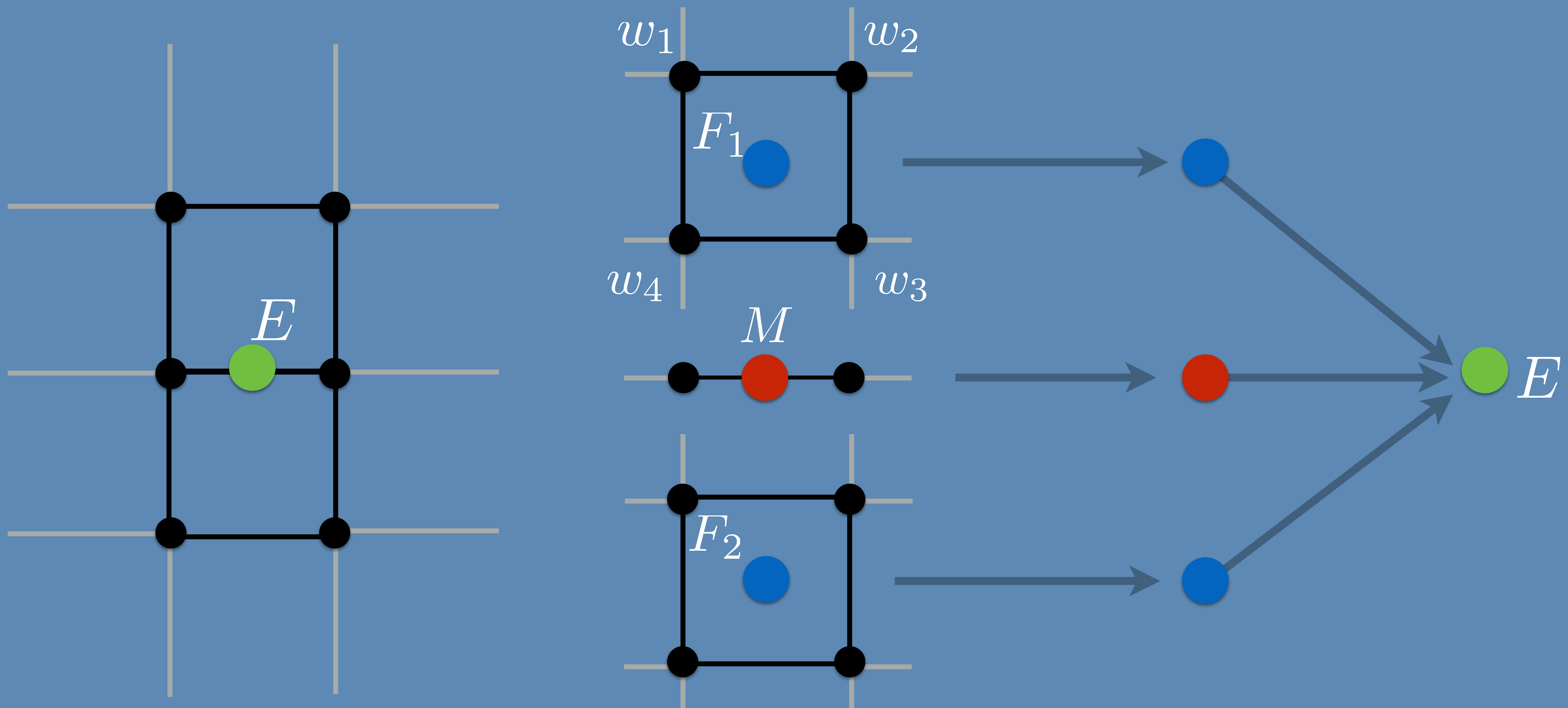

Non-uniform knot intervals

# NURSS




- Non-uniform recursive subdivision surfaces
- No T-vertices
- Generalization of Catmull-Clark with non-uniform knot intervals

# NURSS Factorization



*Tensor-Product Surfaces*

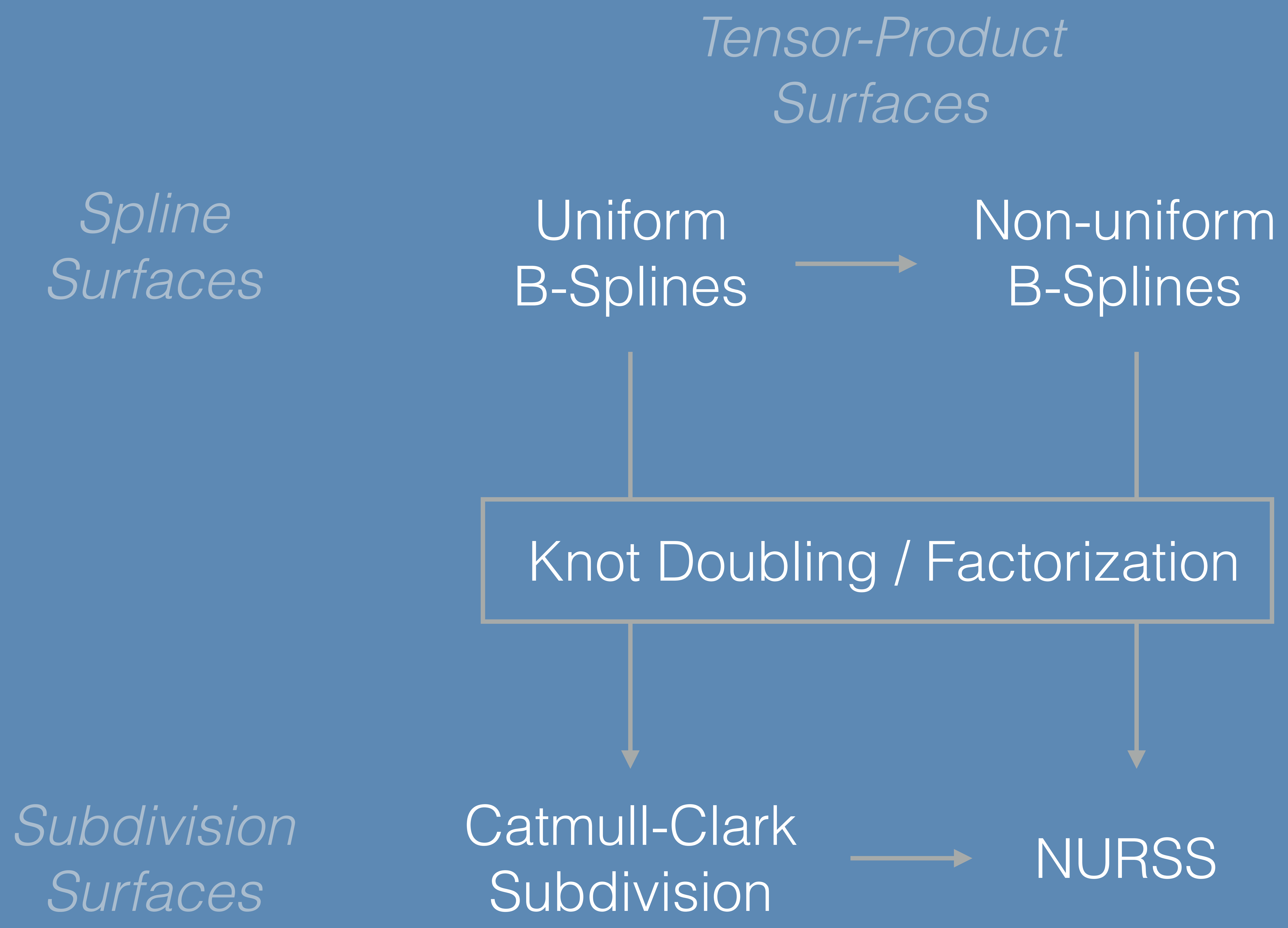
*Spline Surfaces*

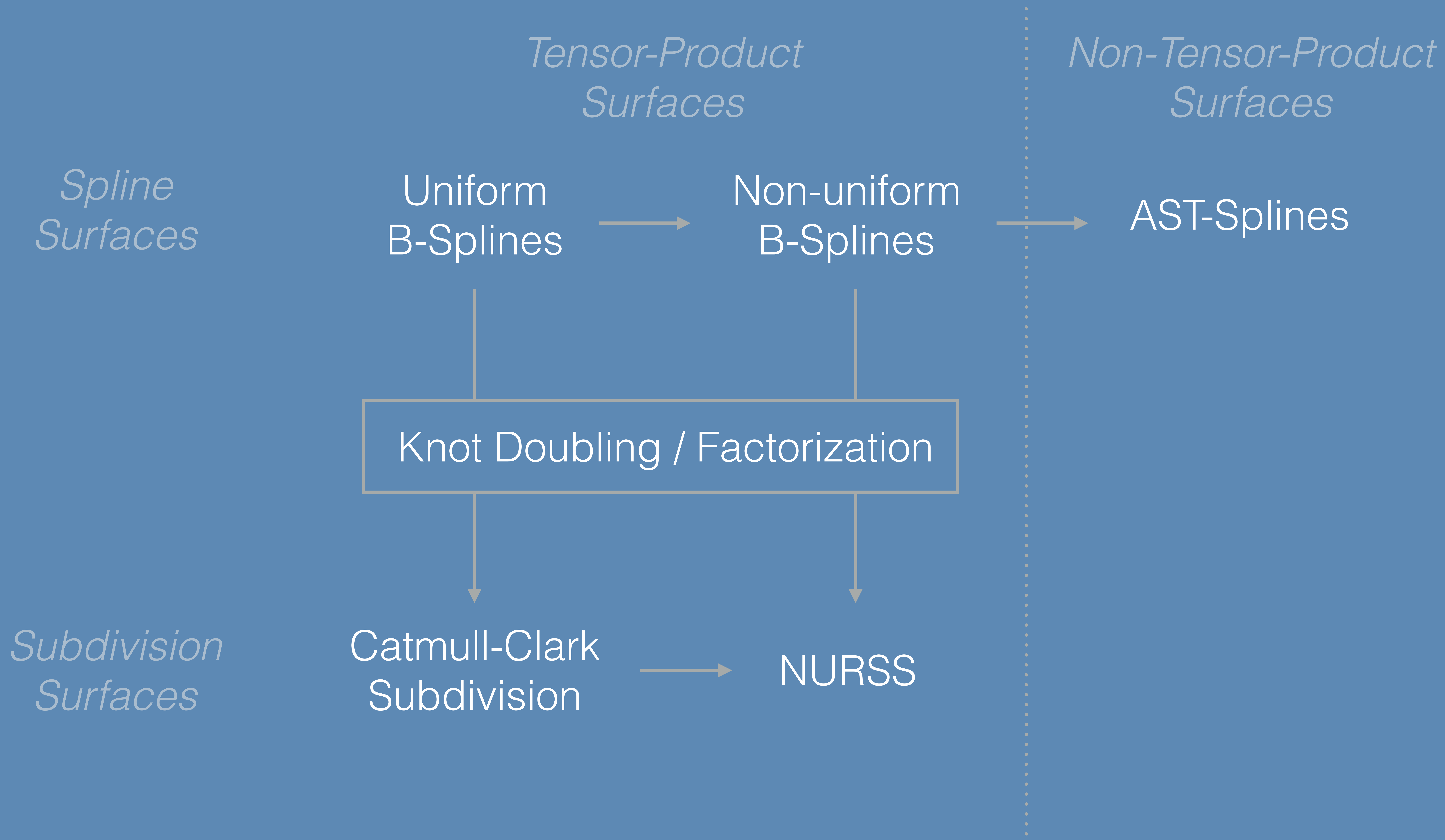
Uniform B-Splines

Knot Doubling / Factorization

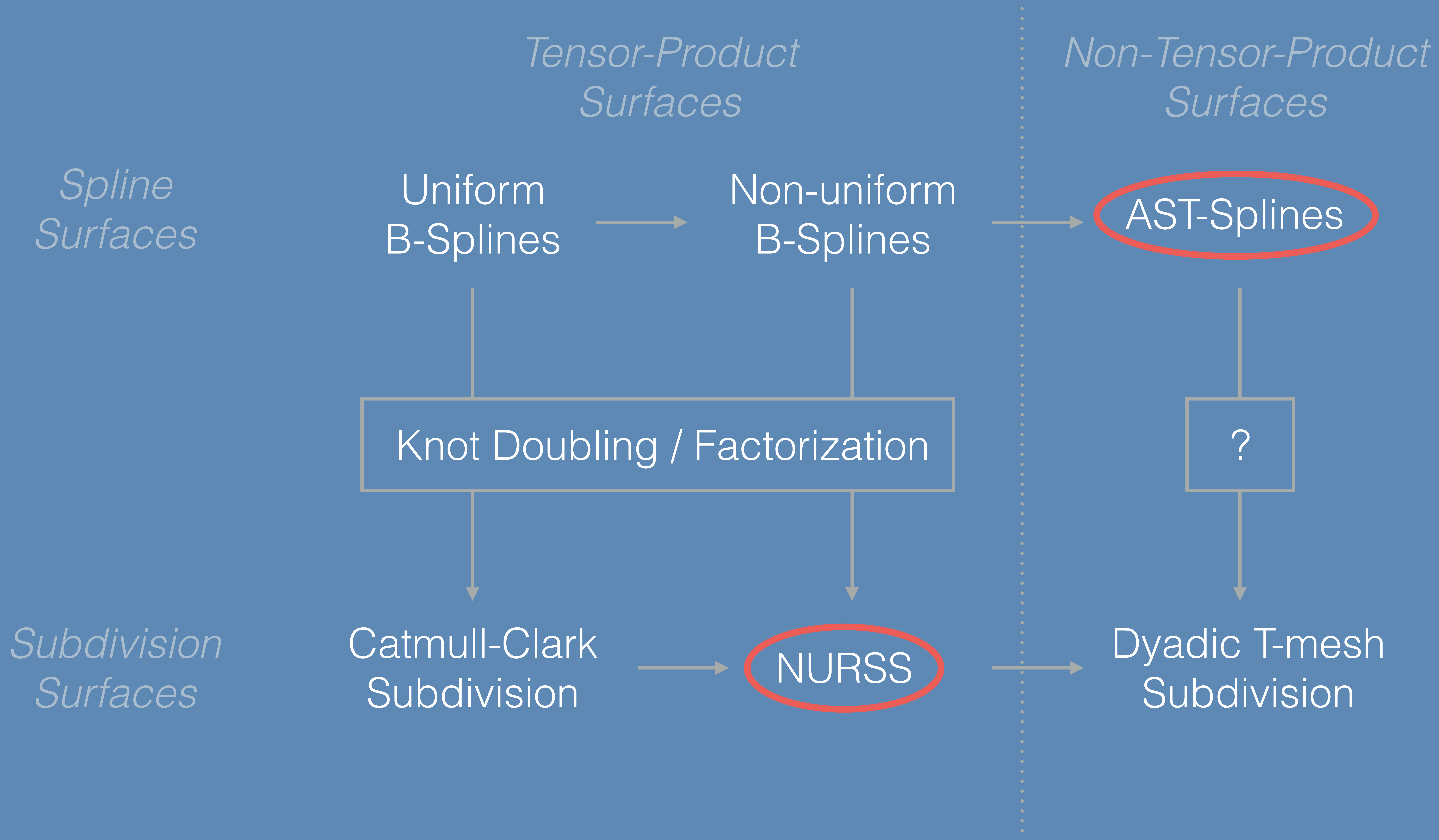
*Subdivision Surfaces*

Catmull-Clark Subdivision


Tensor-Product Surfaces
Spline Surfaces
Uniform B-Splines
Non-uniform B-Splines
Knot Doubling / Factorization
Subdivision Surfaces
Catmull-Clark Subdivision
NURSS

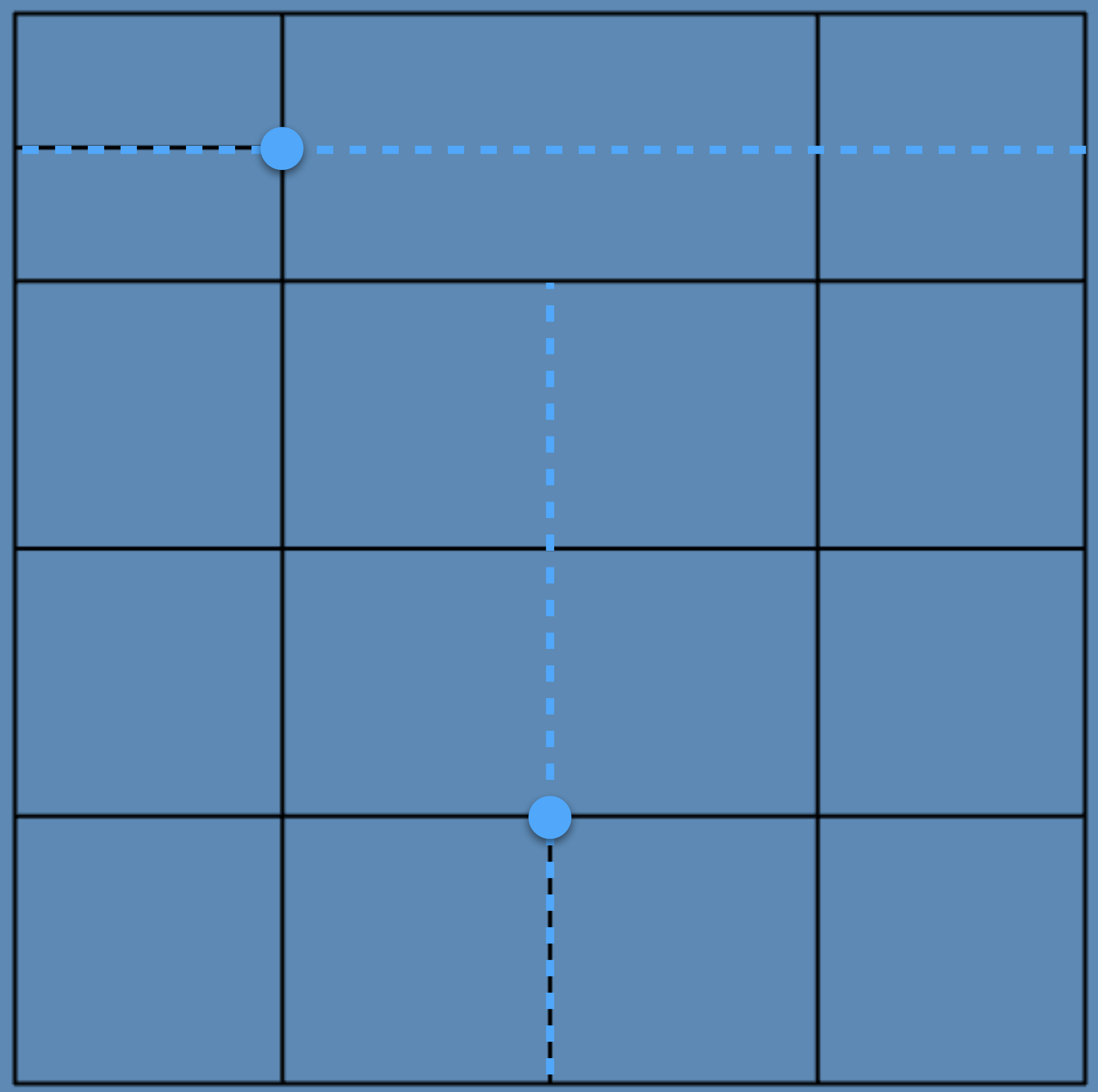


Tensor-Product Surfaces
Non-Tensor-Product Surfaces
Spline Surfaces
Uniform B-Splines
Non-uniform B-Splines
AST-Splines
Knot Doubling / Factorization
Subdivision Surfaces
Catmull-Clark Subdivision
NURSS

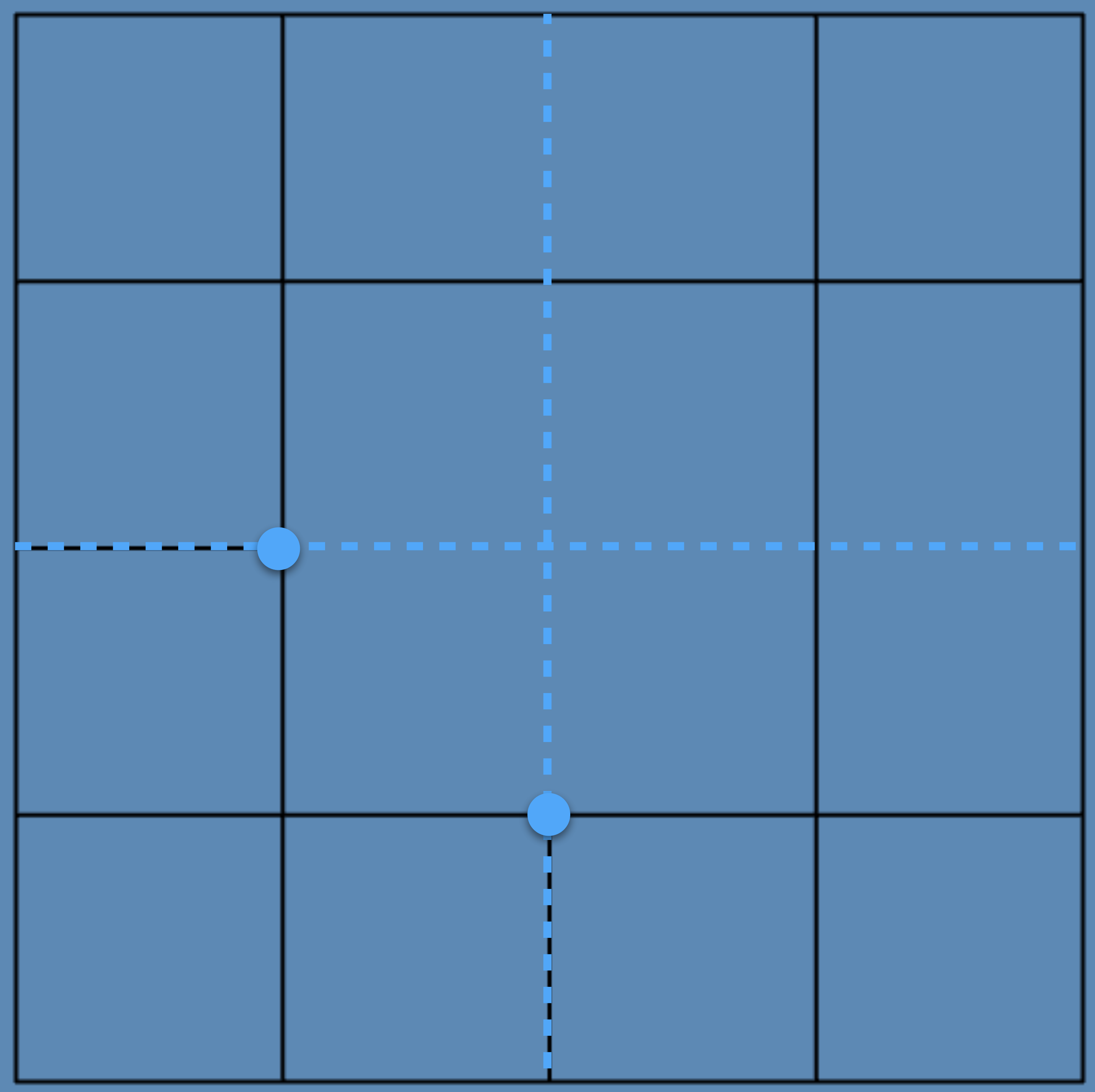


Tensor-Product Surfaces
Non-Tensor-Product Surfaces
Spline Surfaces
Uniform B-Splines
Non-uniform B-Splines
AST-Splines
Knot Doubling / Factorization
?
Subdivision Surfaces
Catmull-Clark Subdivision
NURSS
Dyadic T-mesh Subdivision

# For Which T-meshes Is Knot Doubling Possible?



Analysis-Suitable



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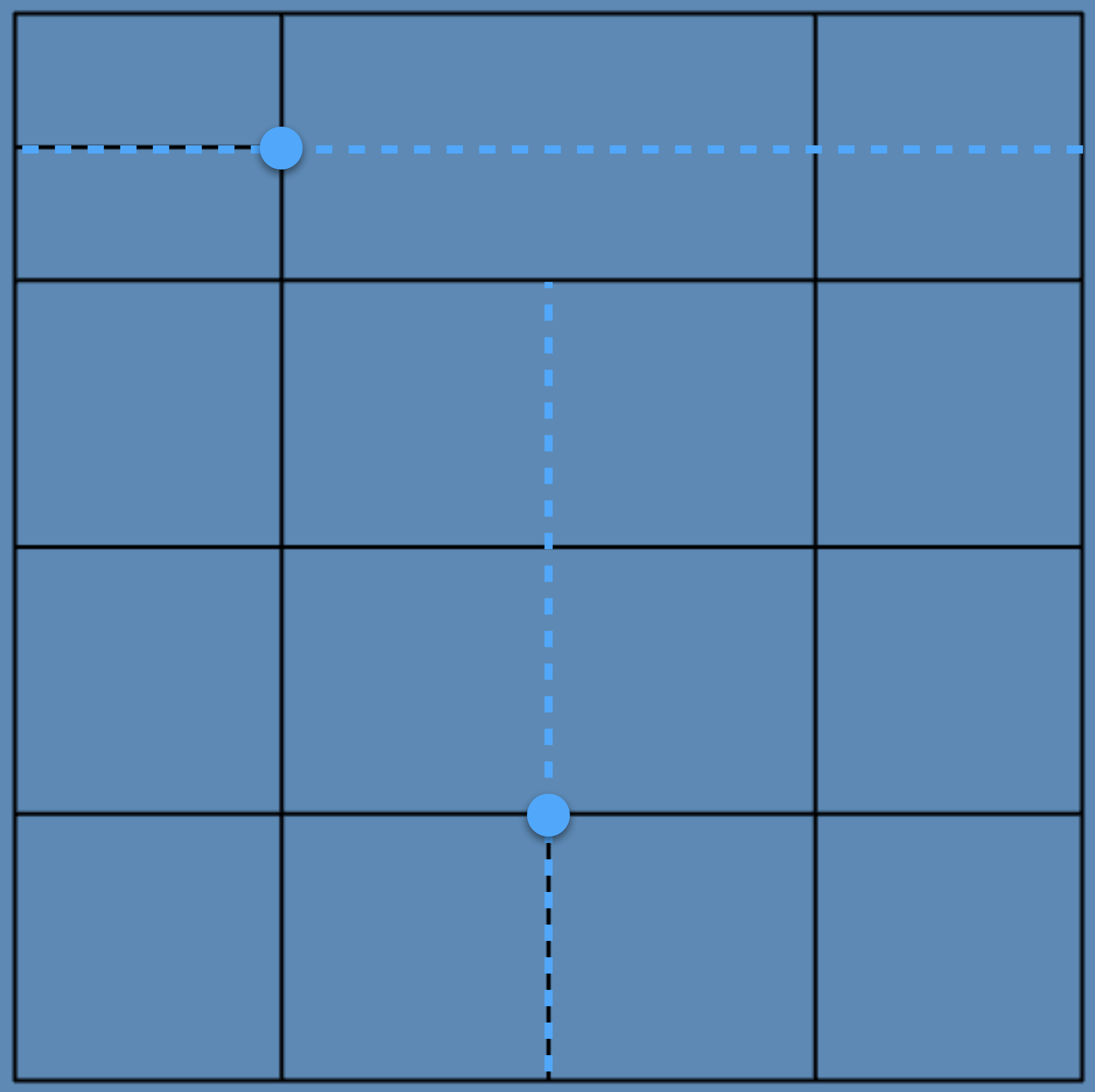
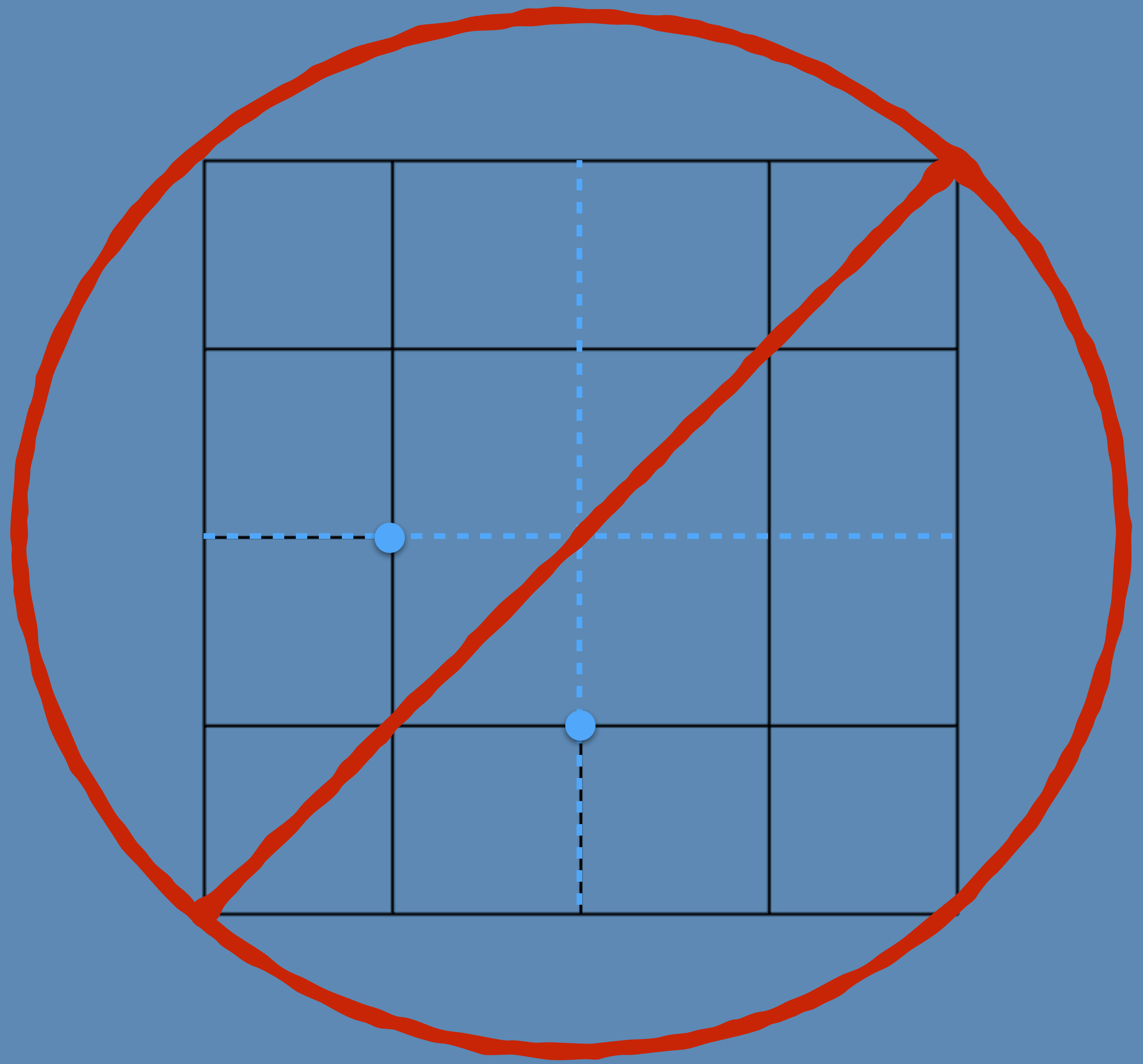


Analysis-Suitable



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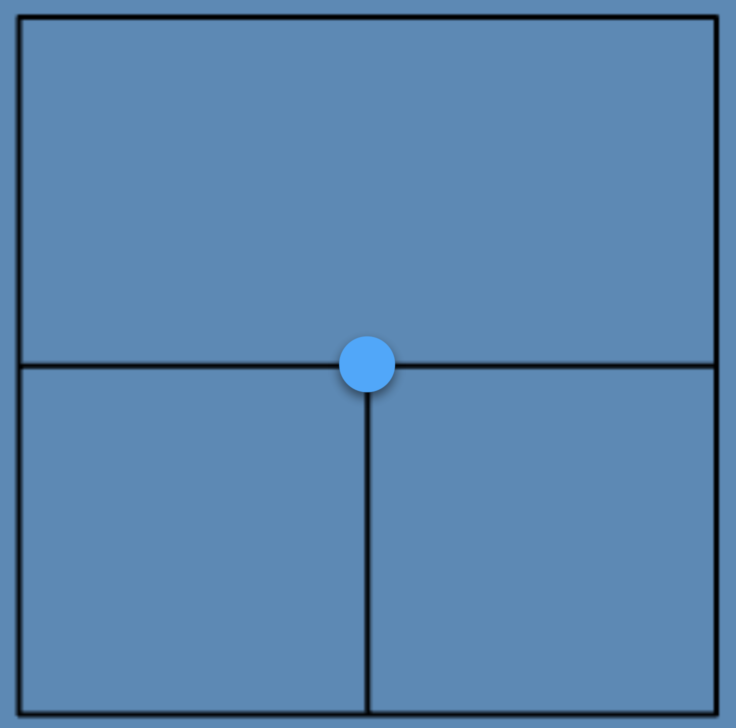
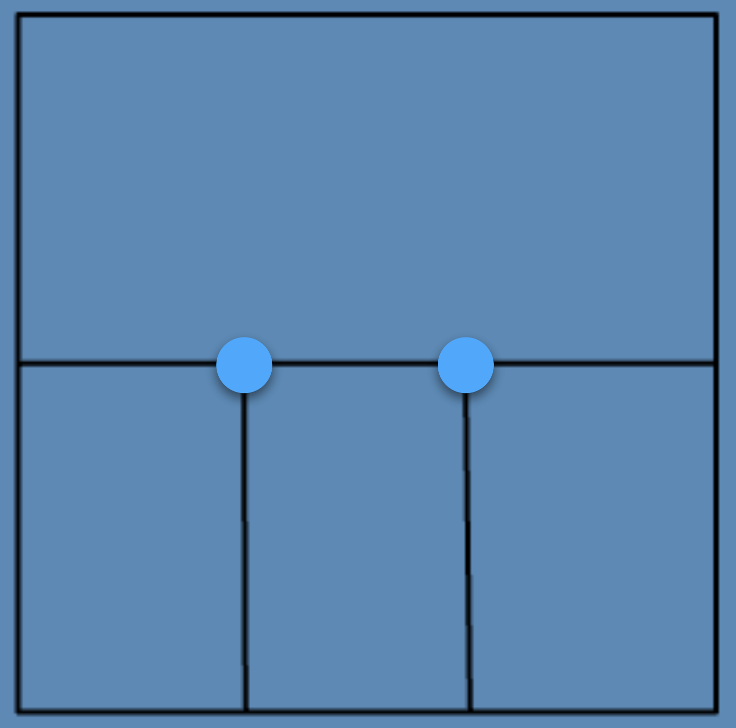
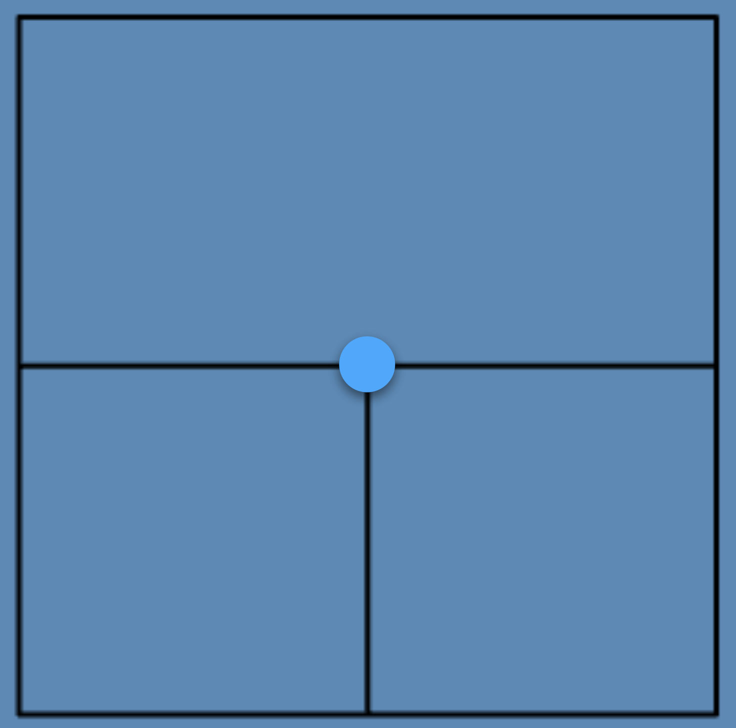


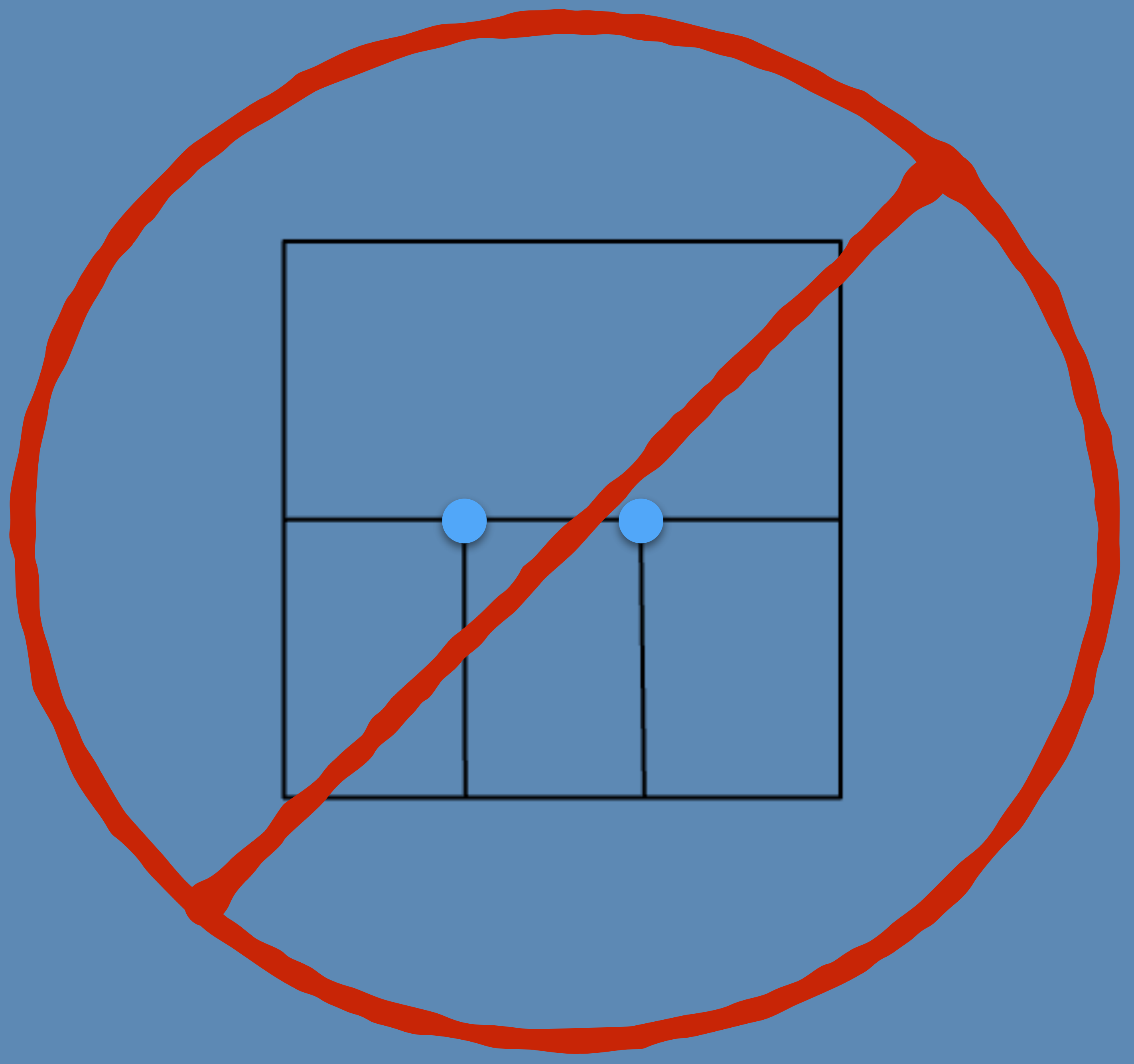
Dyadic



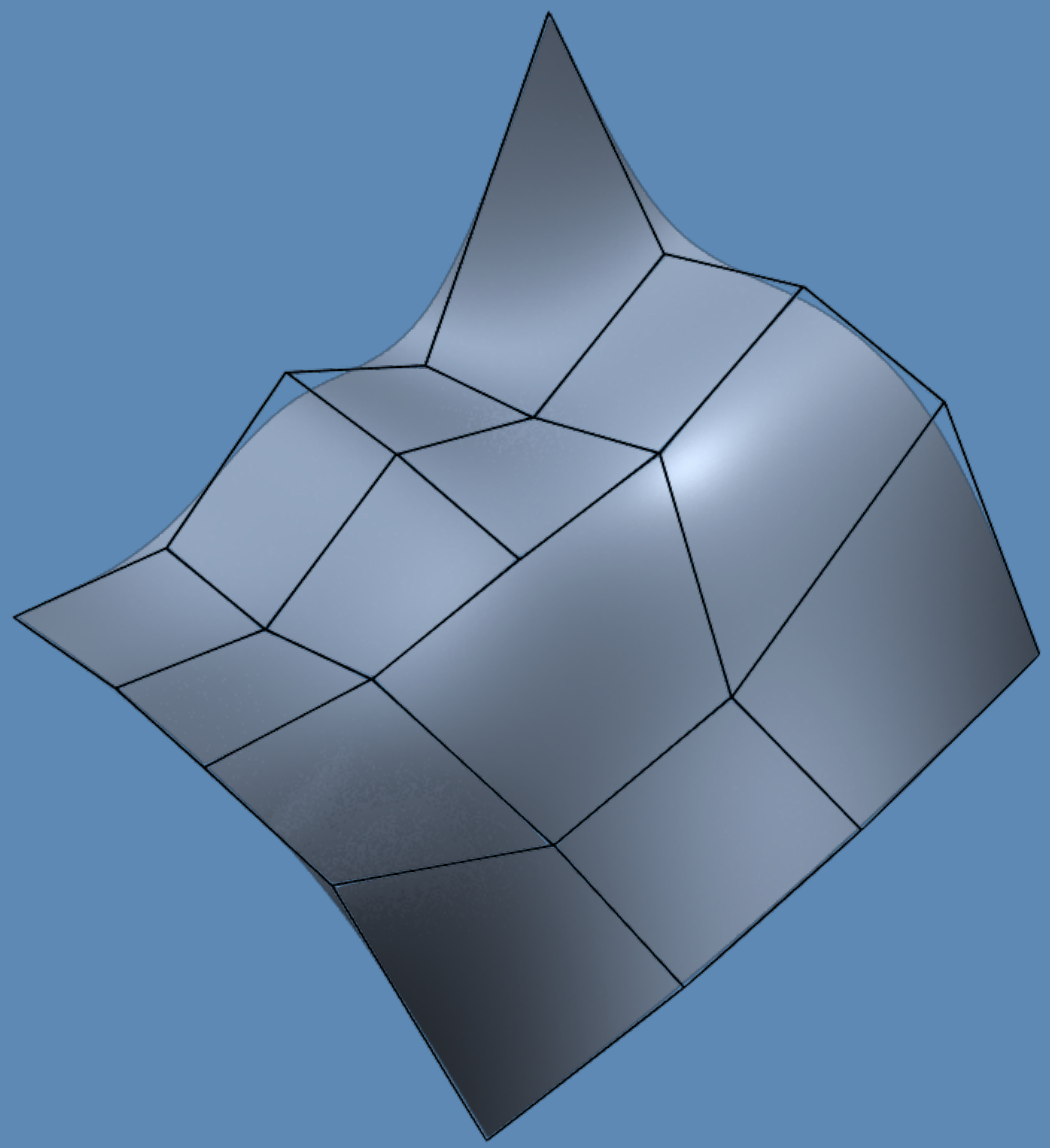
# For Which T-meshes Is Knot Doubling Possible?



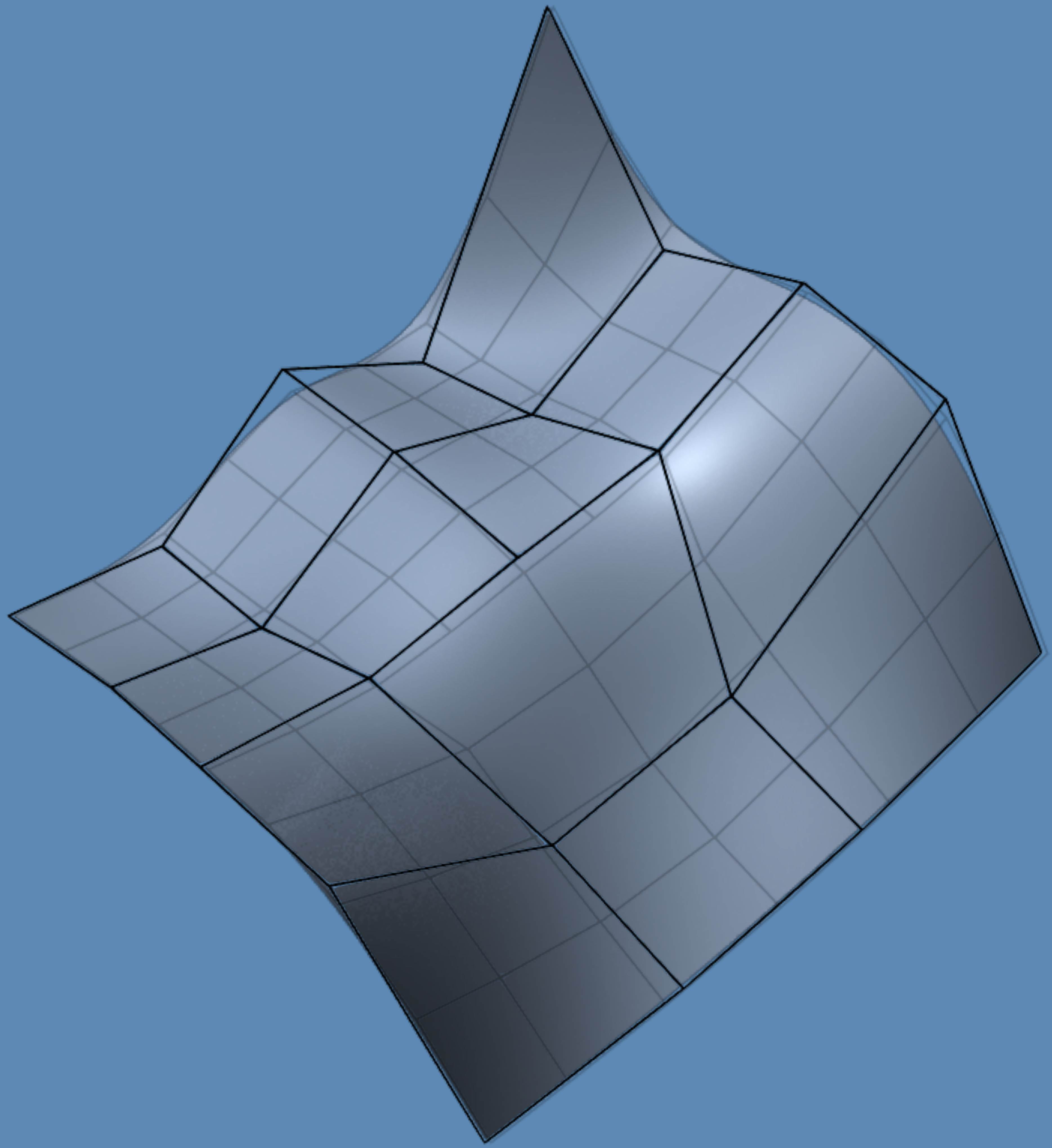
Dyadic



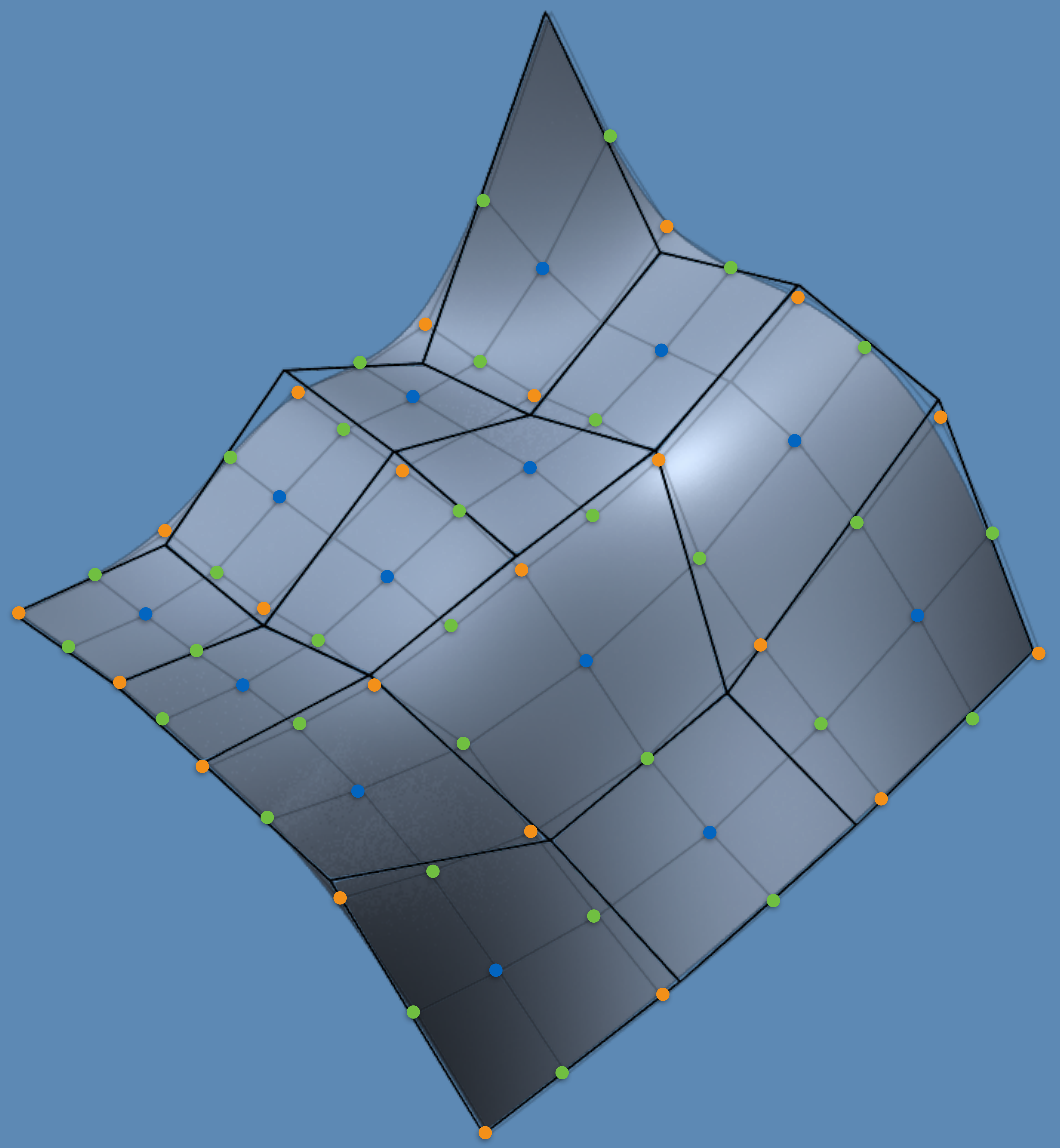
# Dyadic AST-Spline Knot Doubling



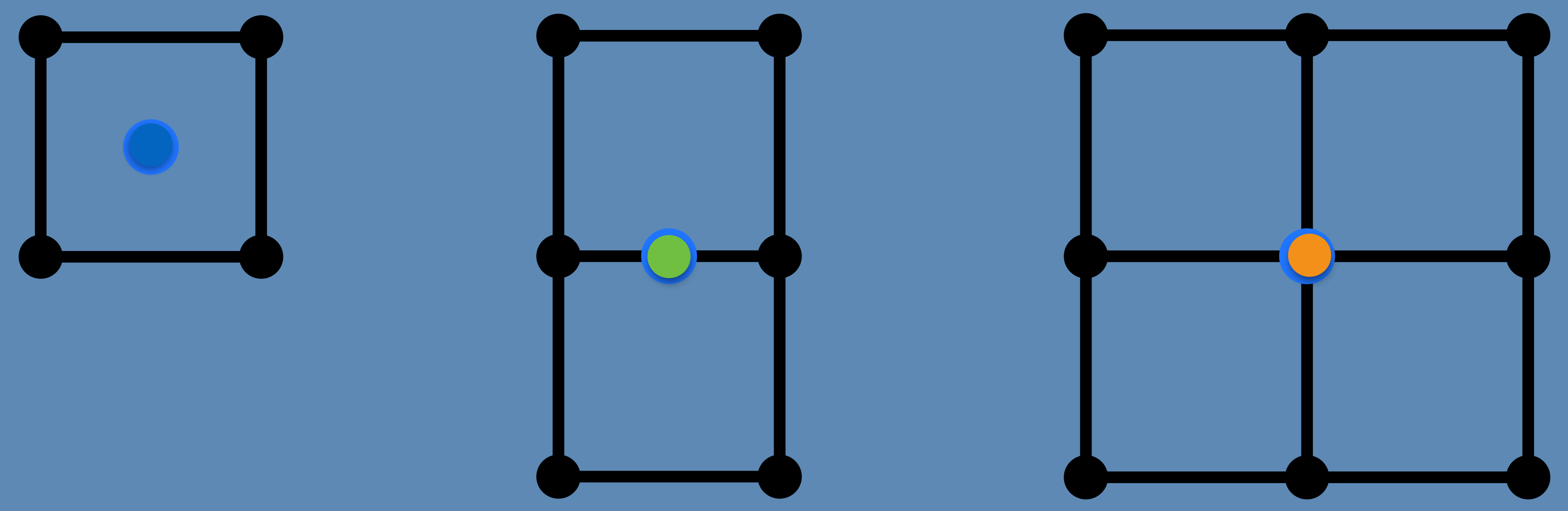
# Dyadic AST-Spline Knot Doubling



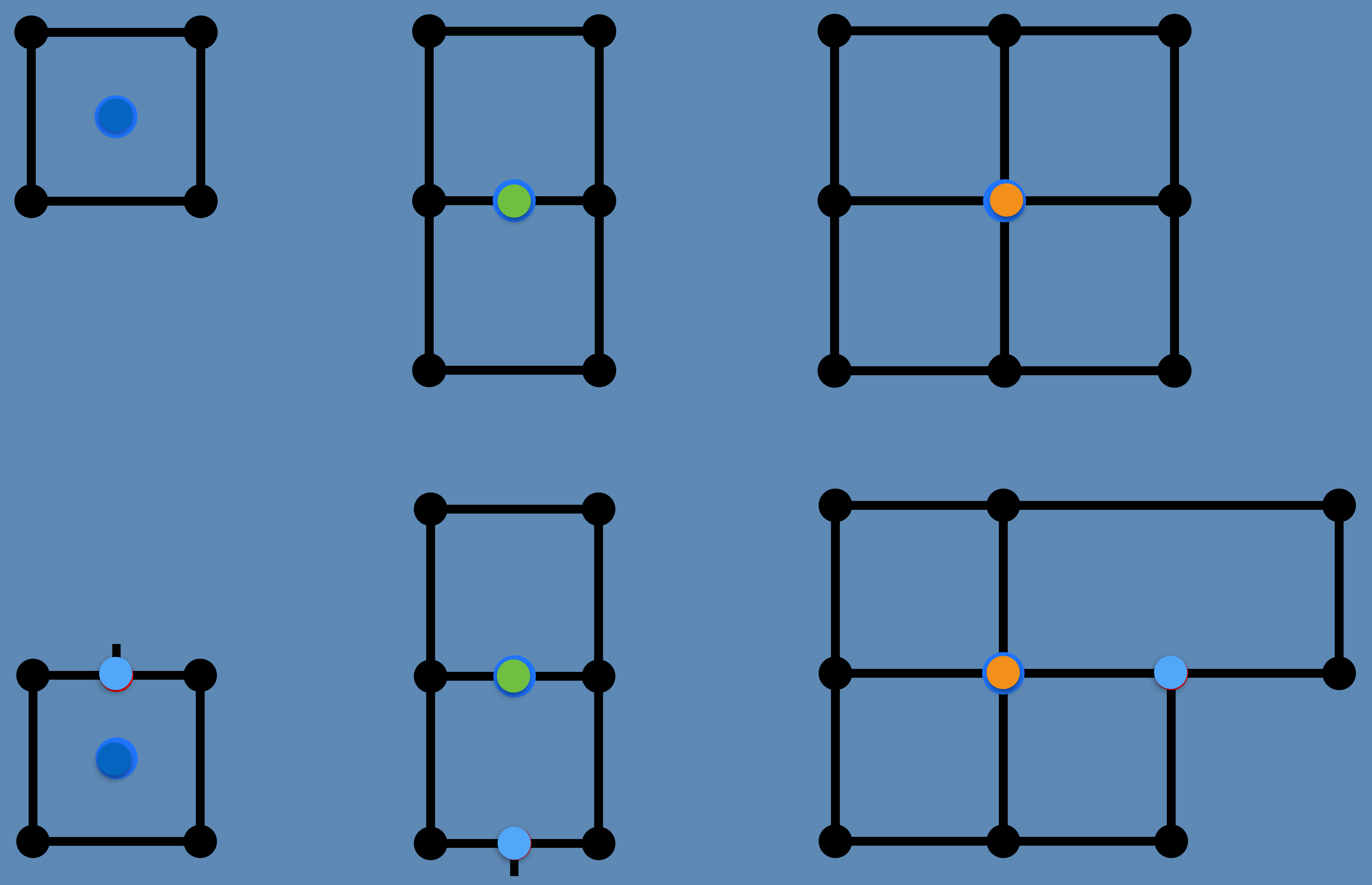
# Dyadic AST-Spline Knot Doubling



# Generalizing Factorization to T-meshes



# Generalizing Factorization to T-meshes



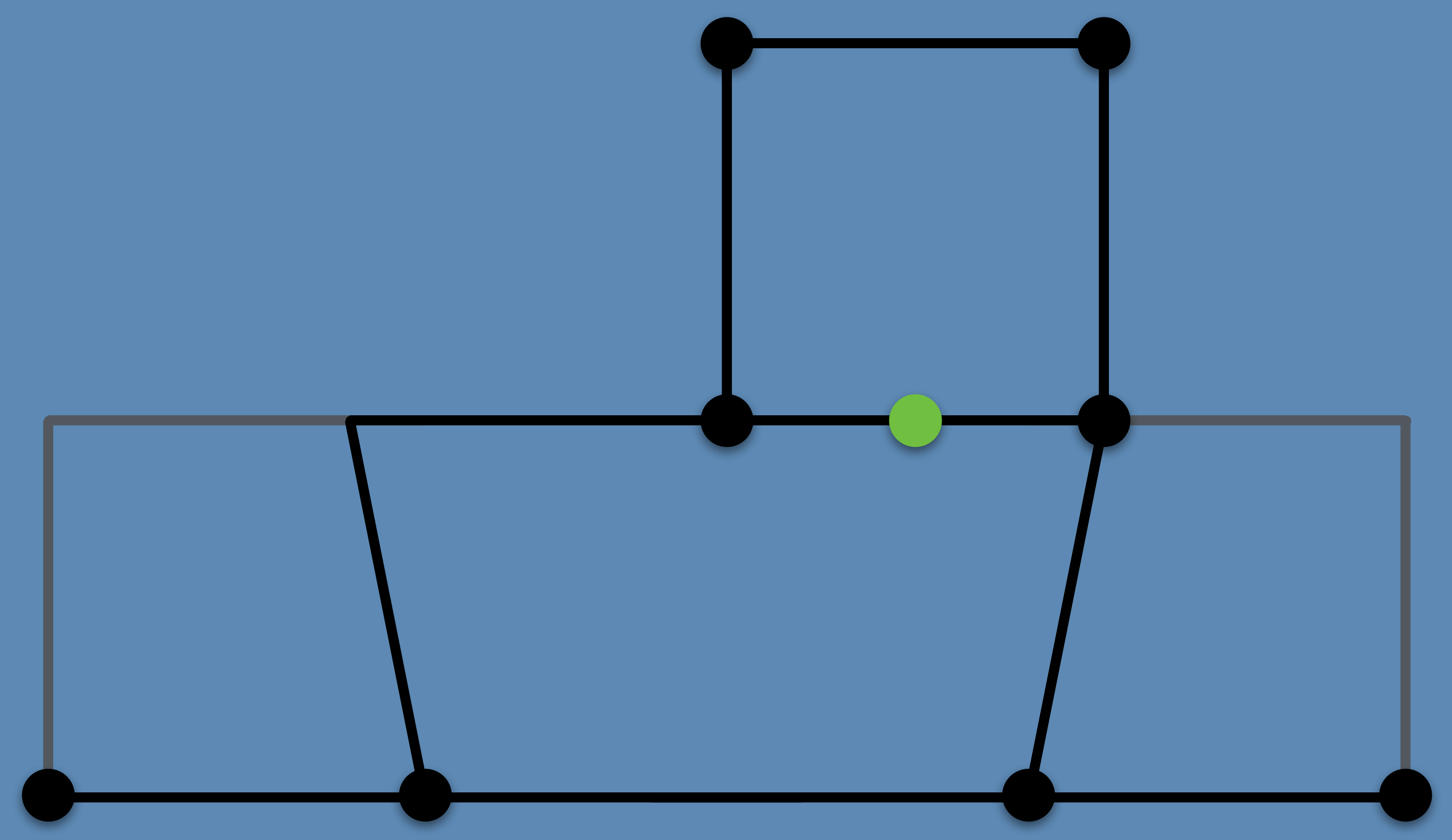
# Generalizing Factorization to T-meshes

# Generalizing Factorization to T-meshes

1. Regularizing stencils

2. T-joint weight adjustments

# 1) Regularizing stencils



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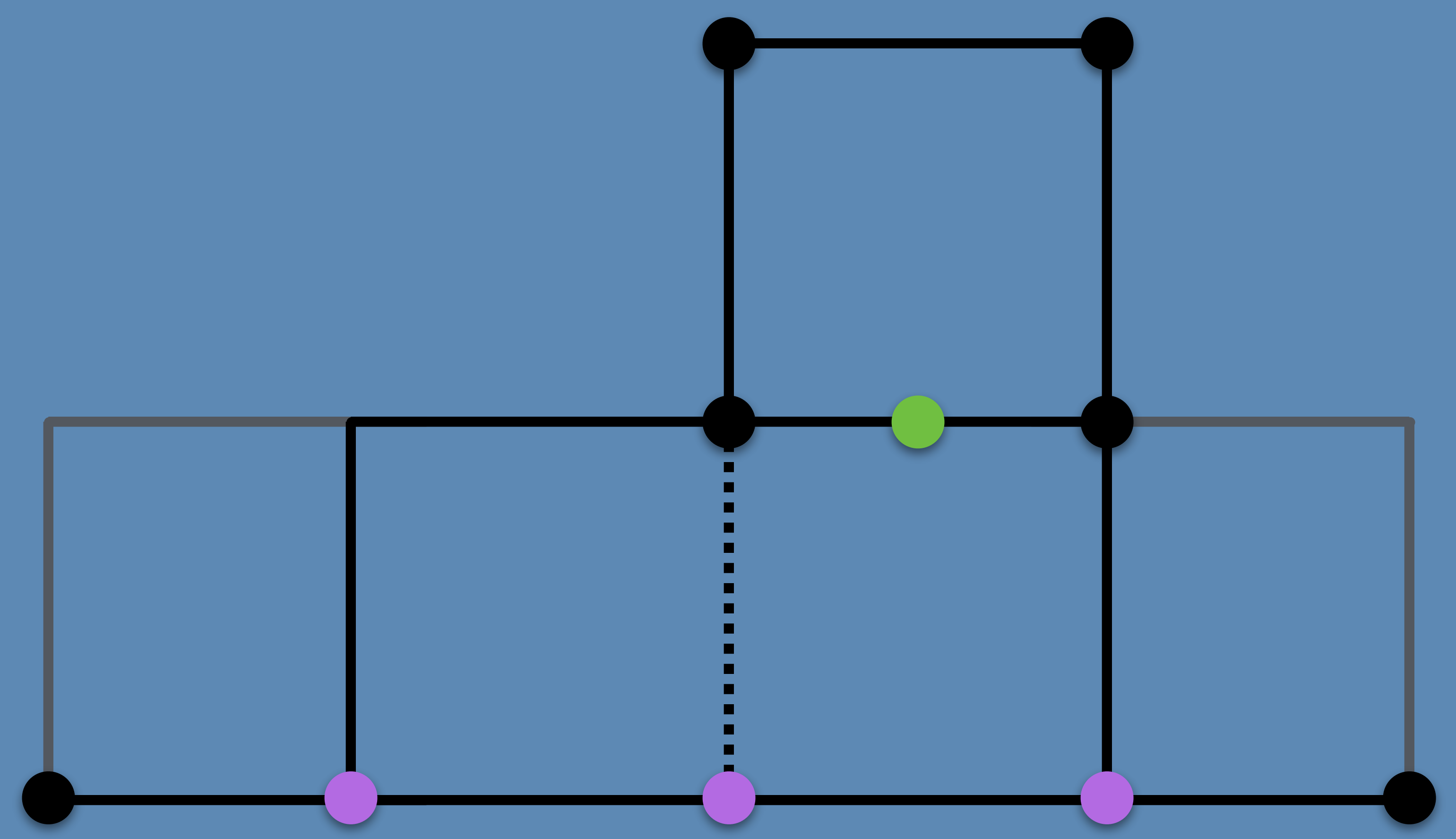

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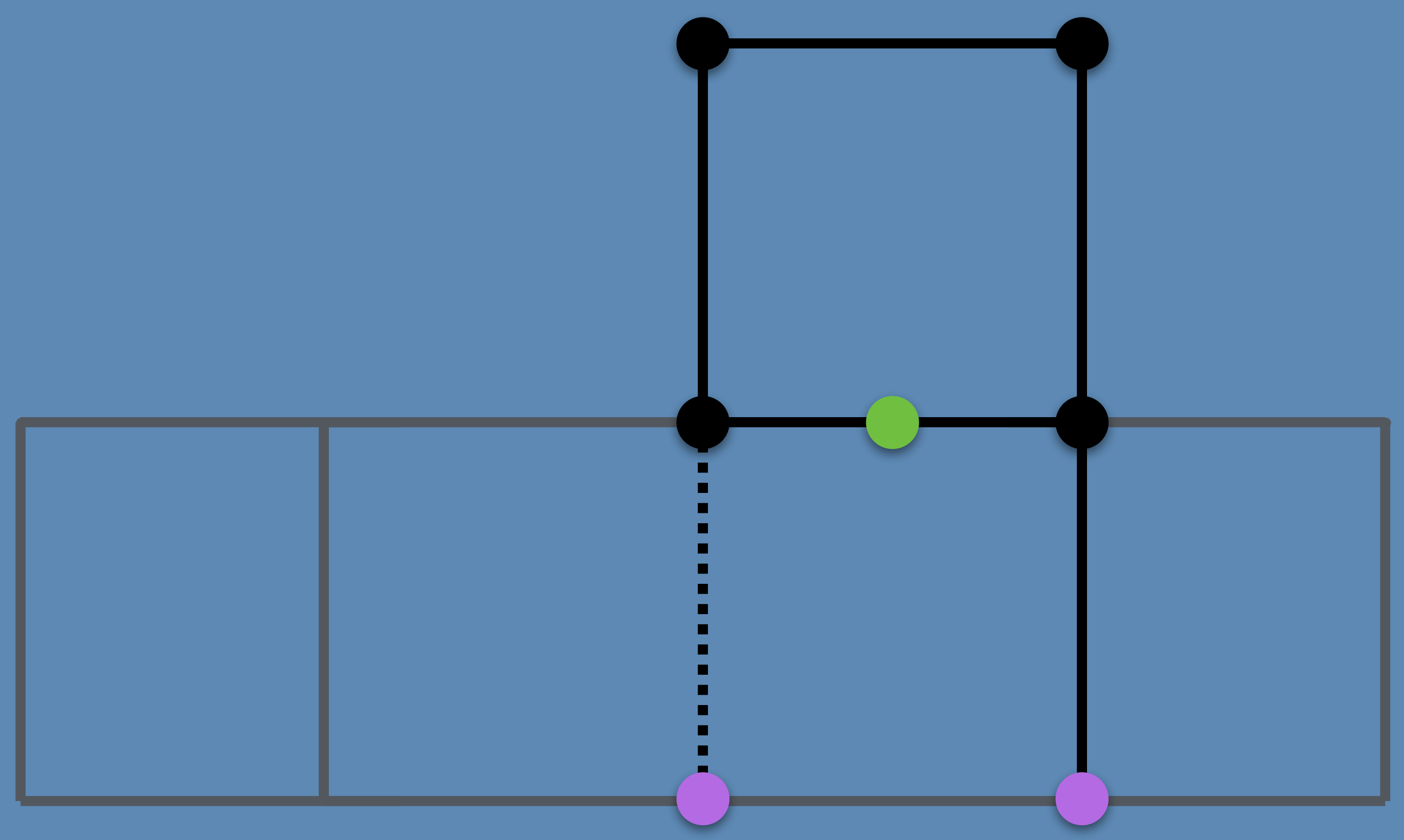

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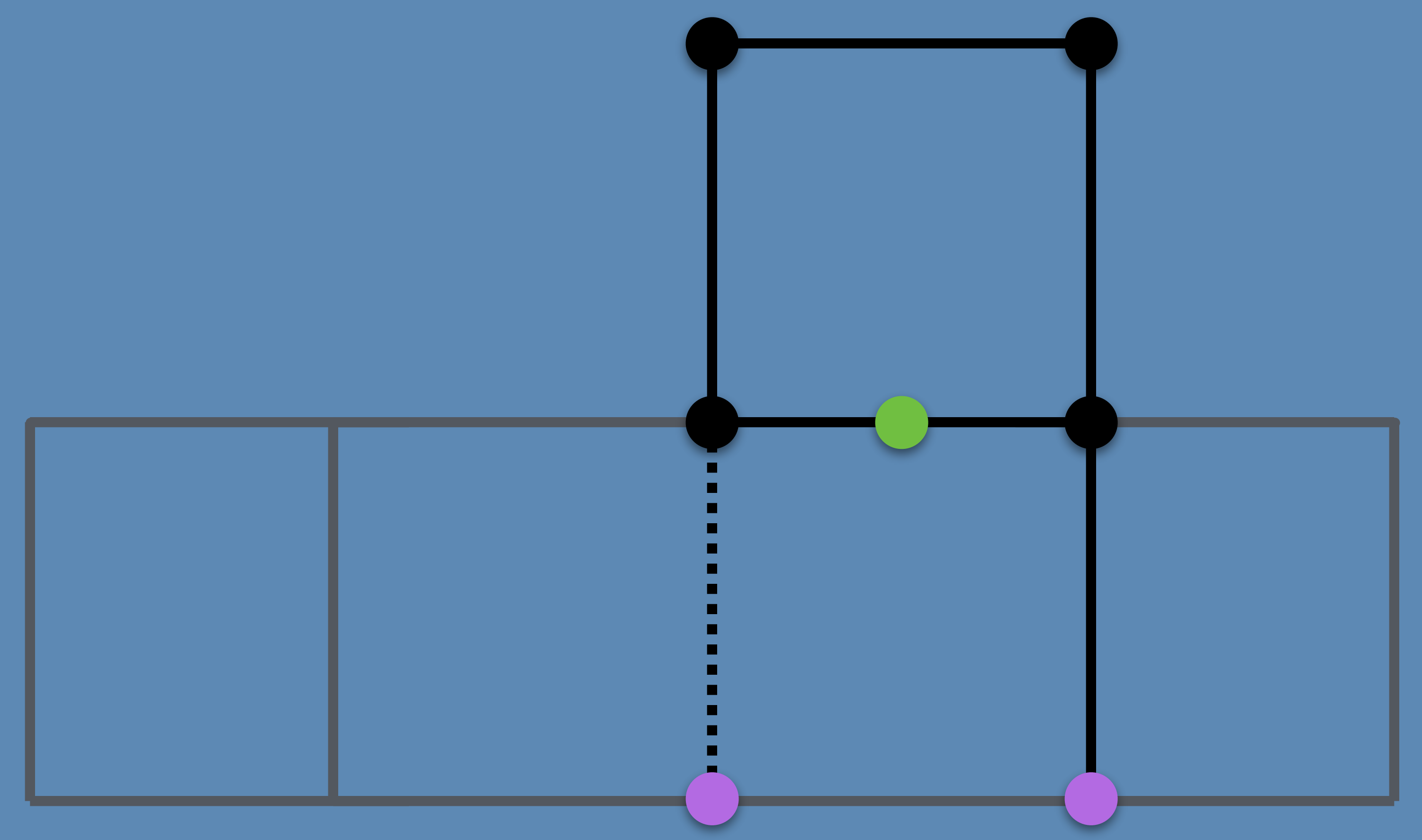

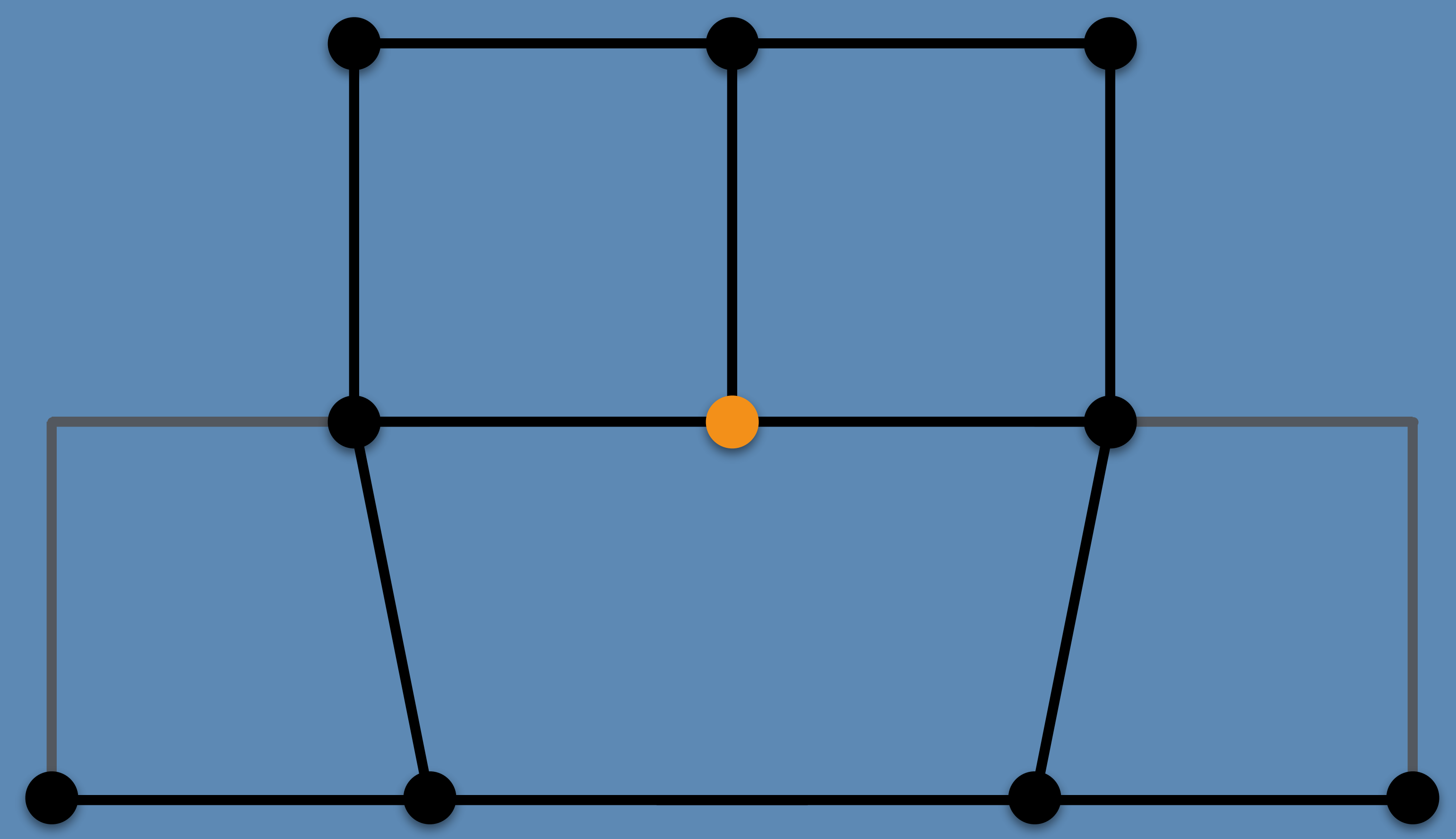

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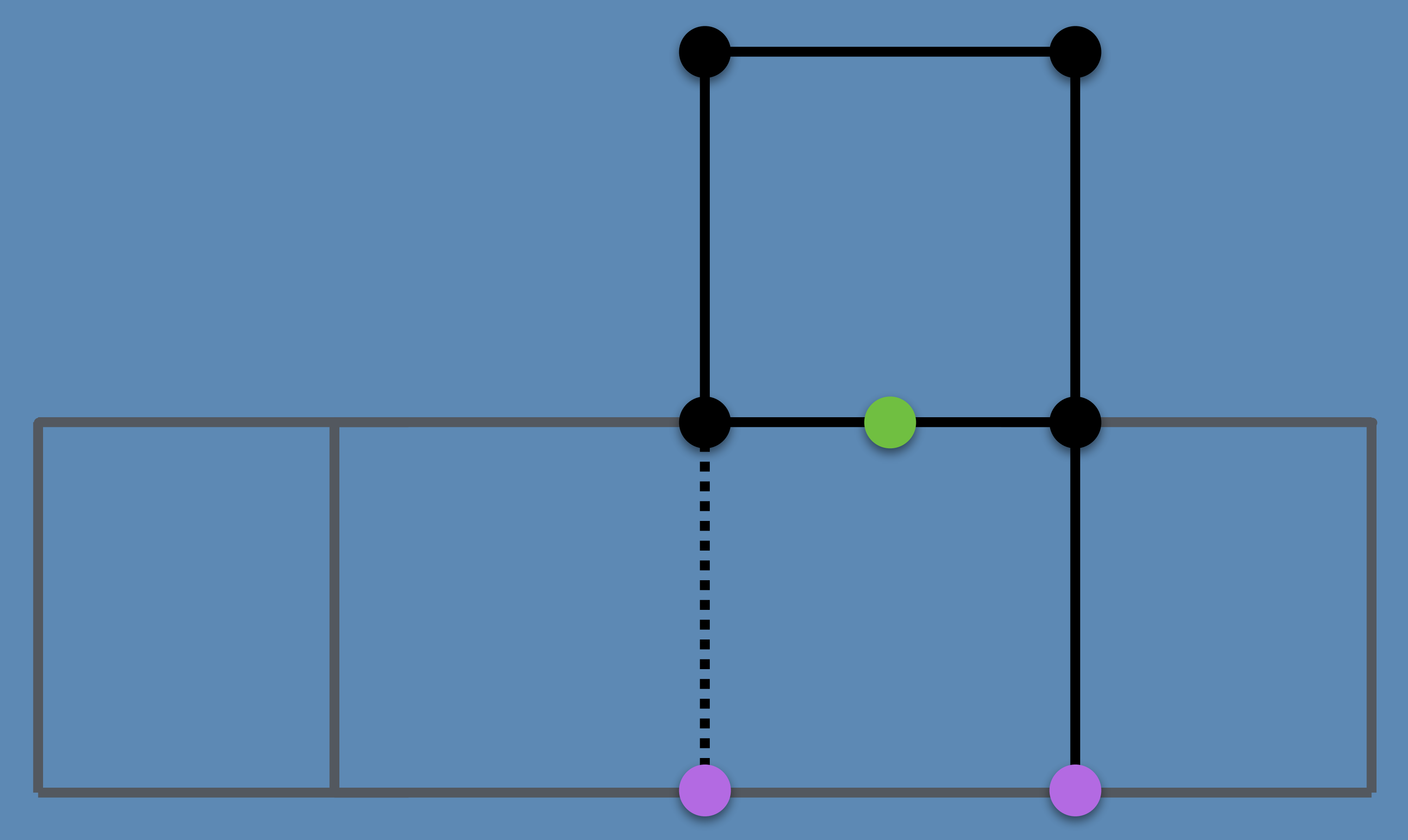
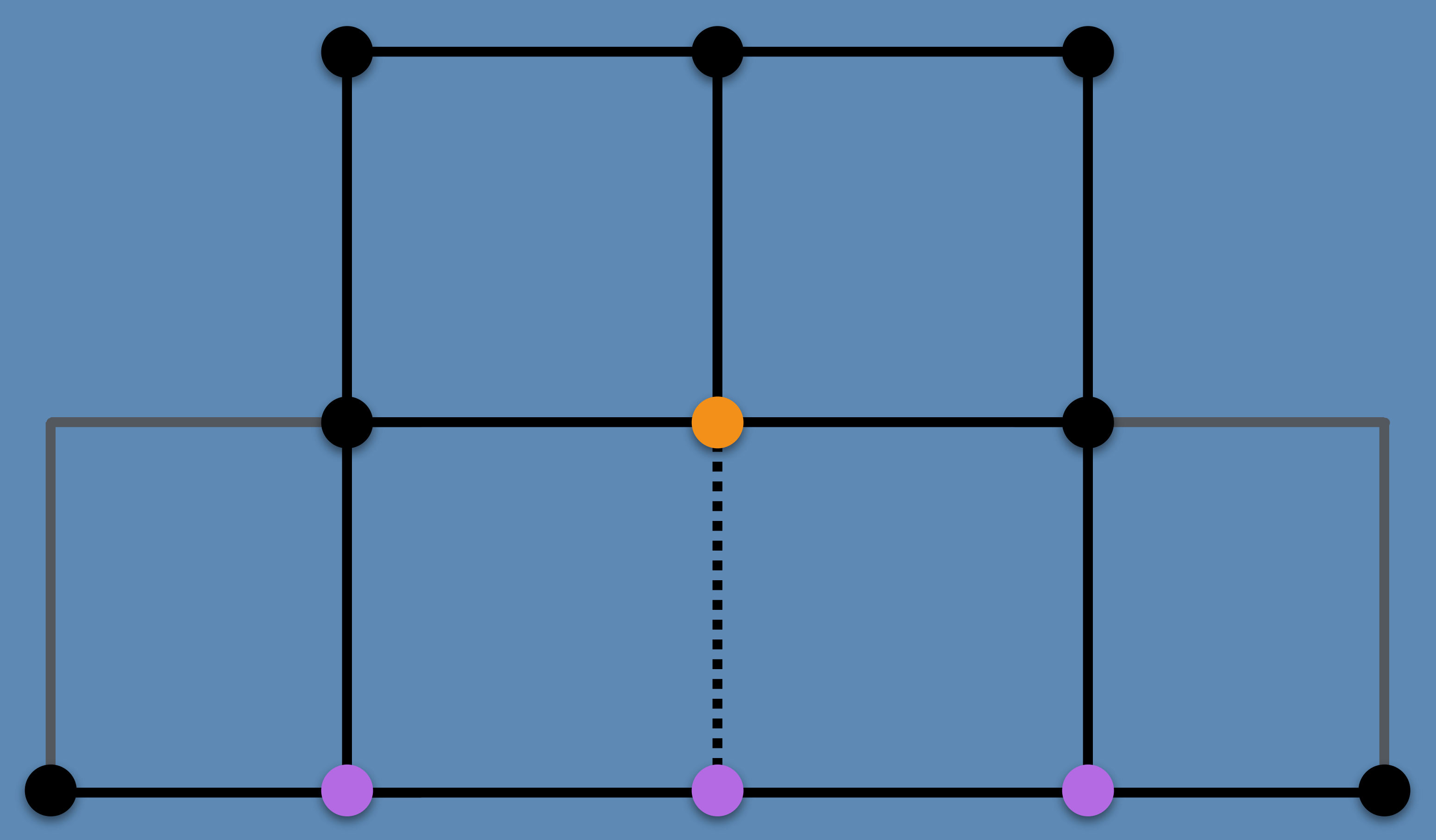
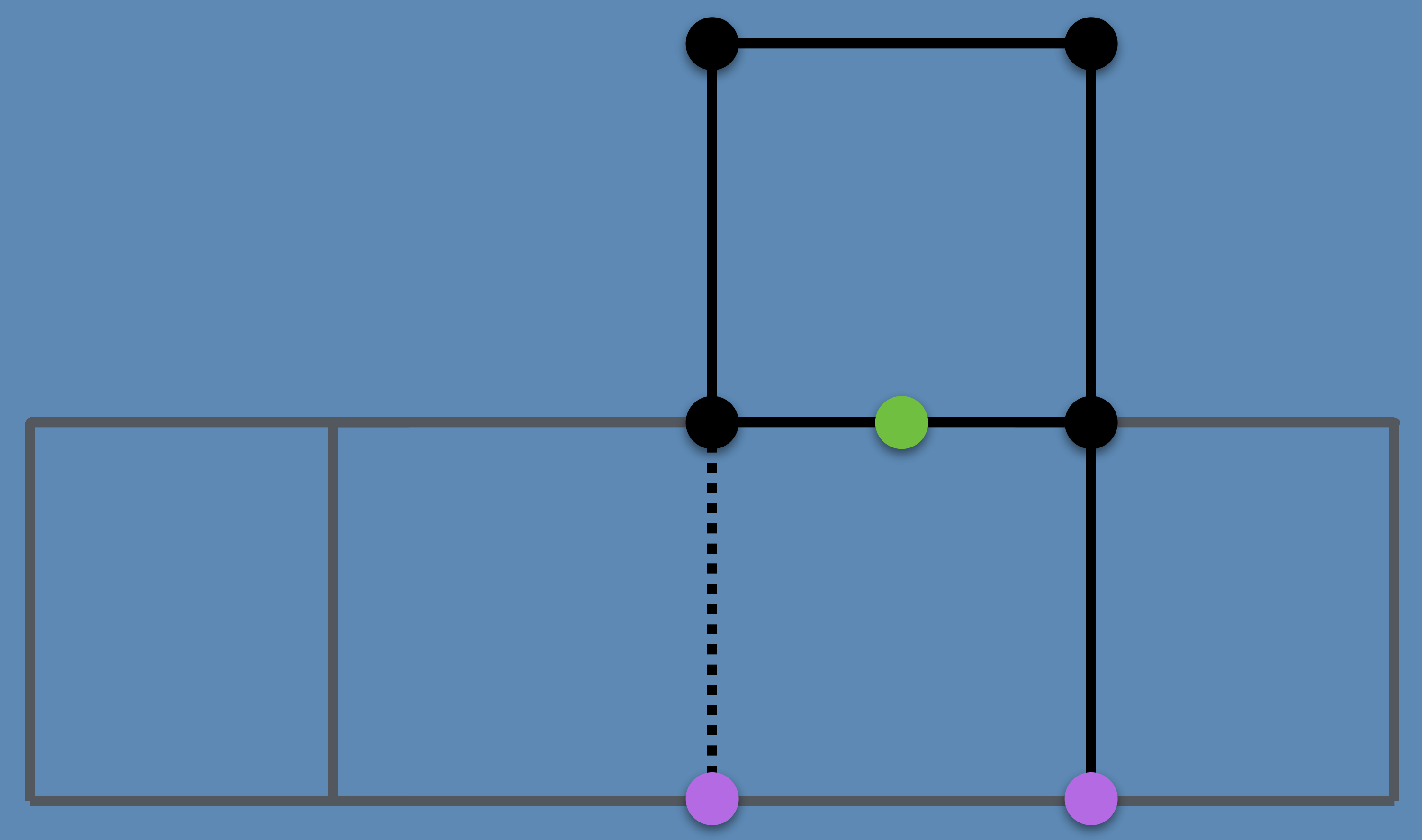
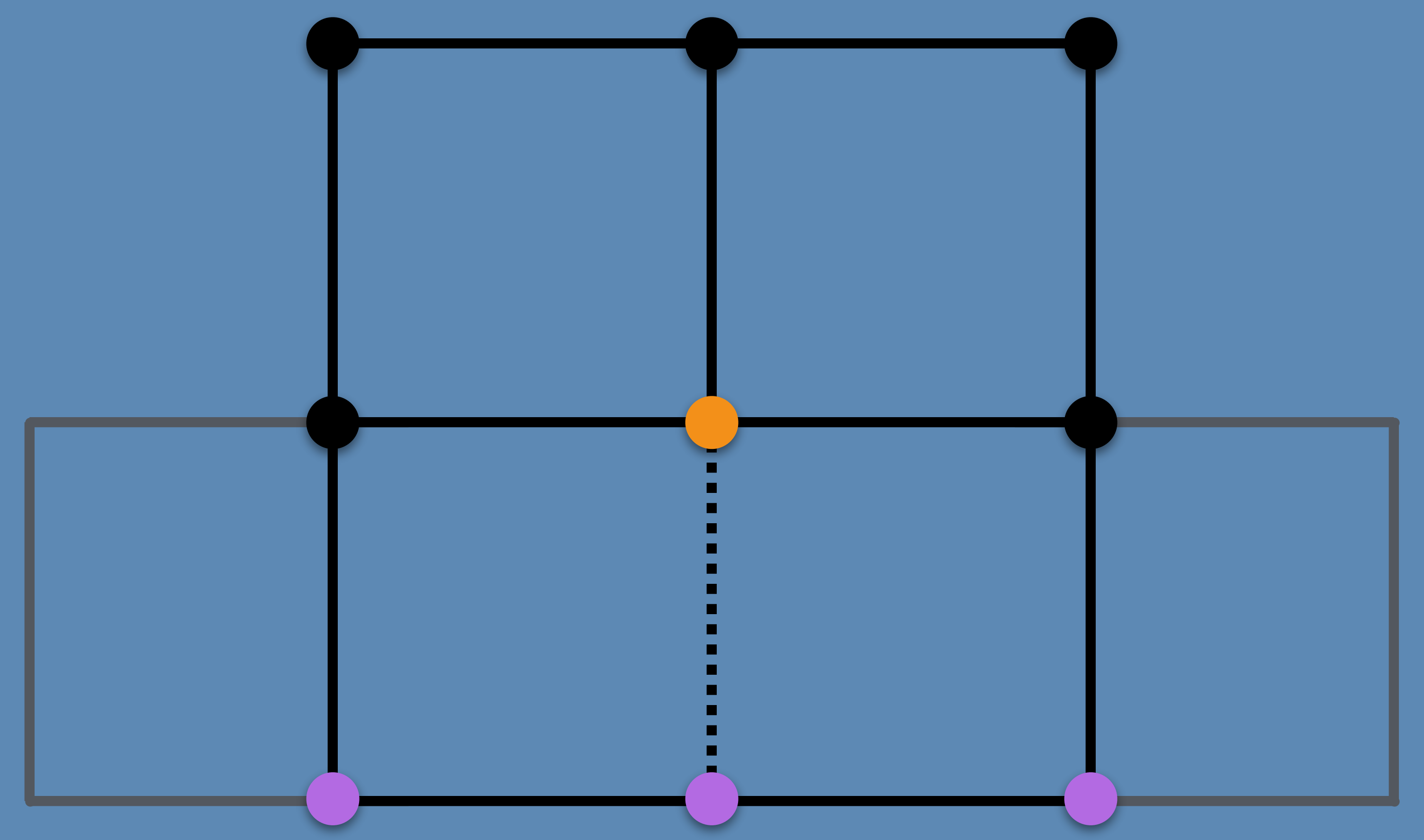




# 1) Regularizing stencils

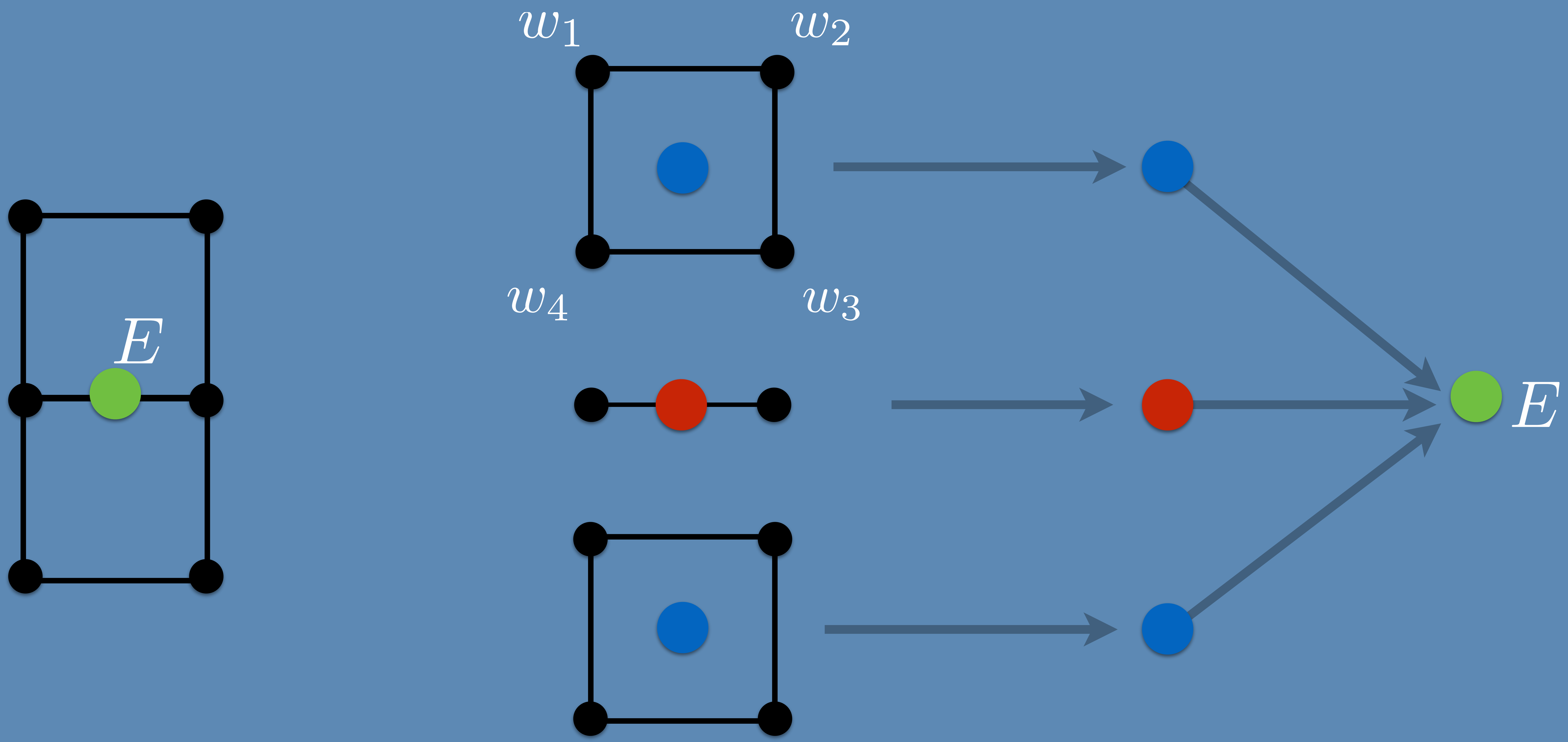




Close, but not quite

# 2) T-joint weight adjustments

- T-joints in the stencil result in weights shifted to different vertices



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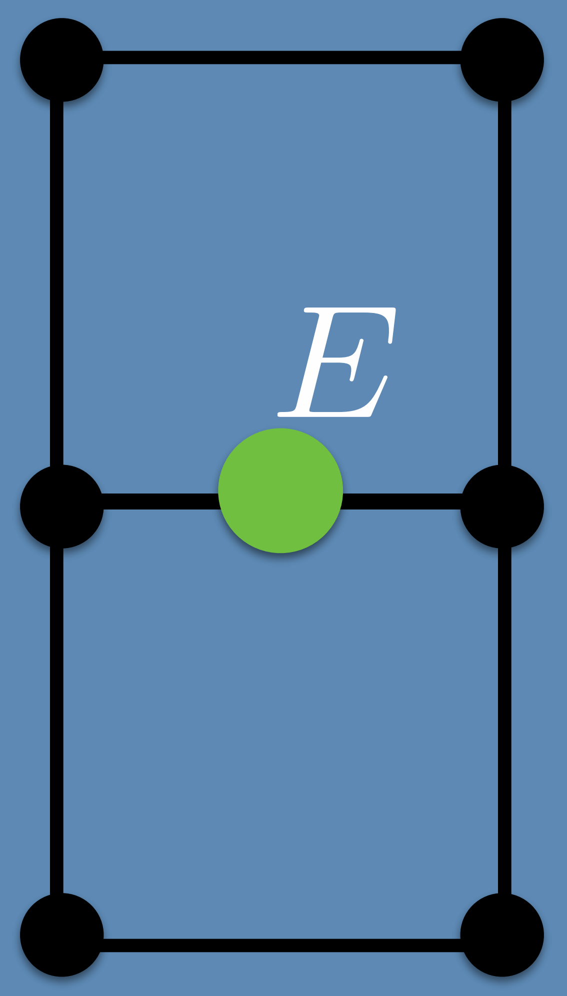


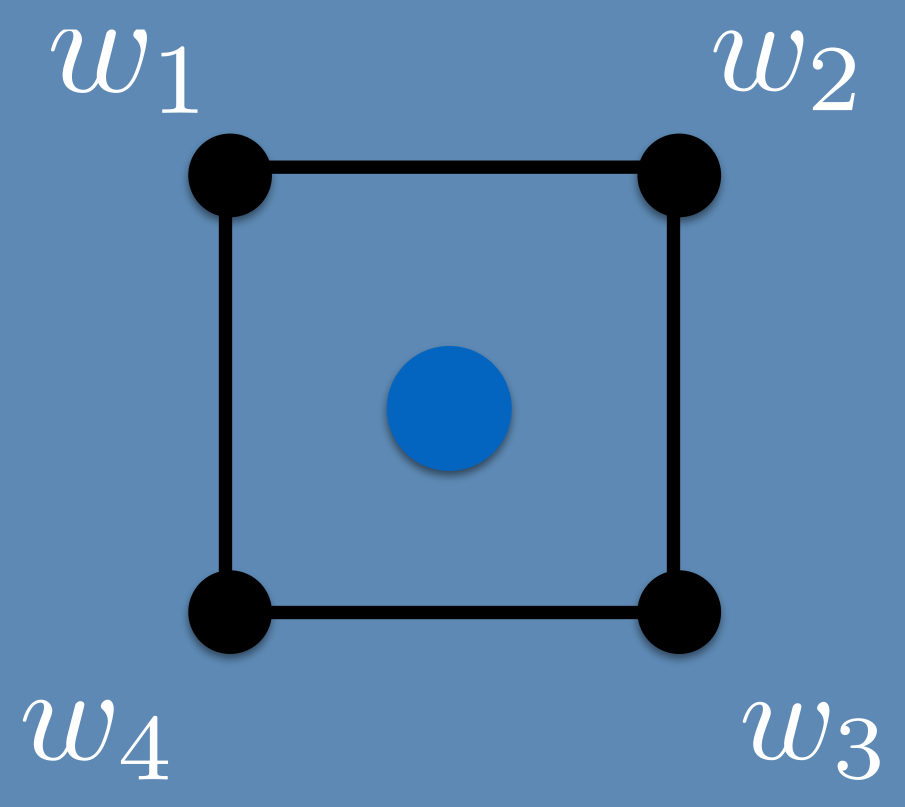

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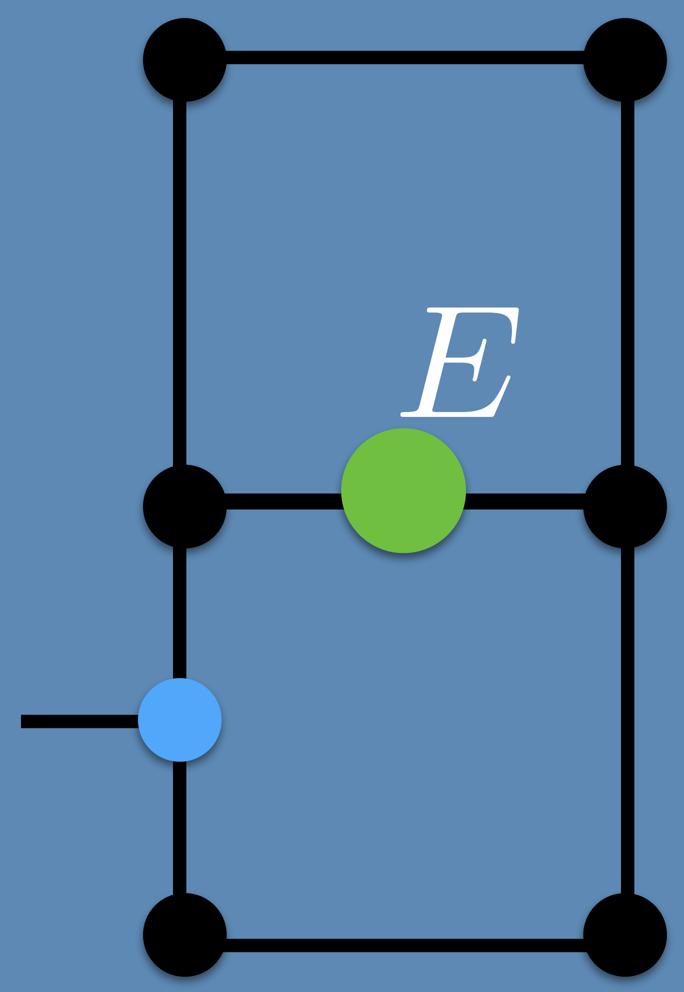


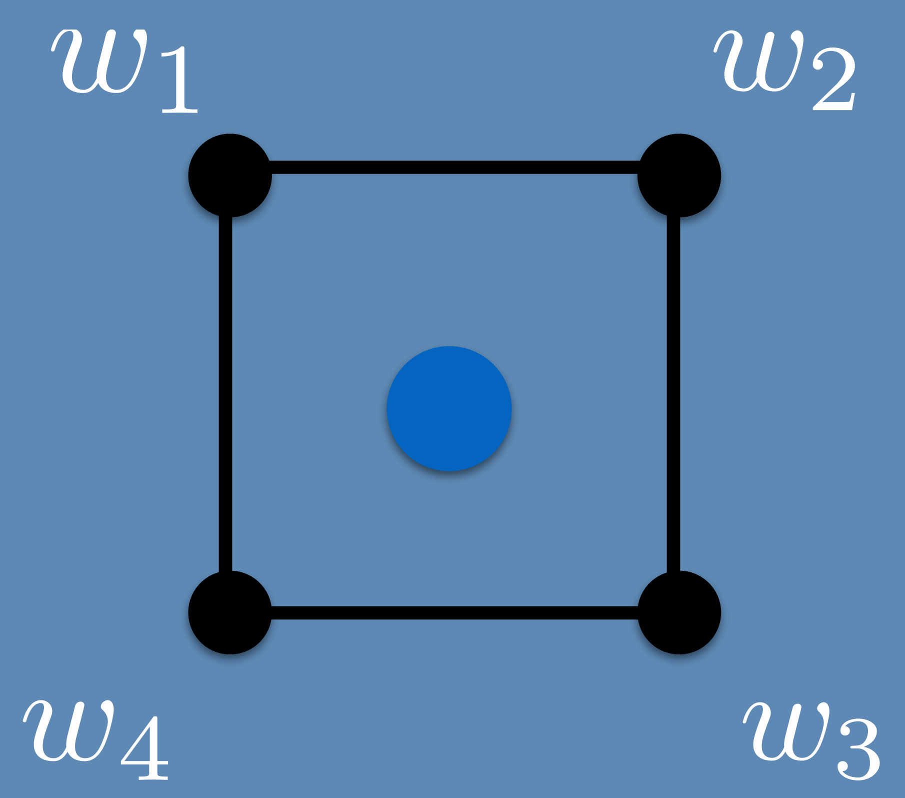

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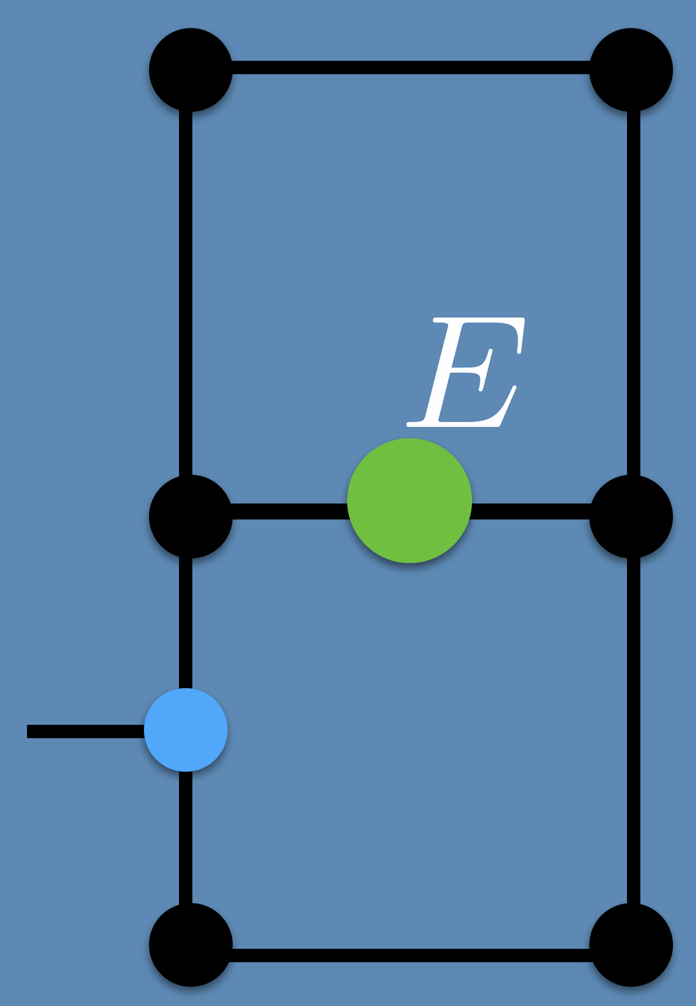


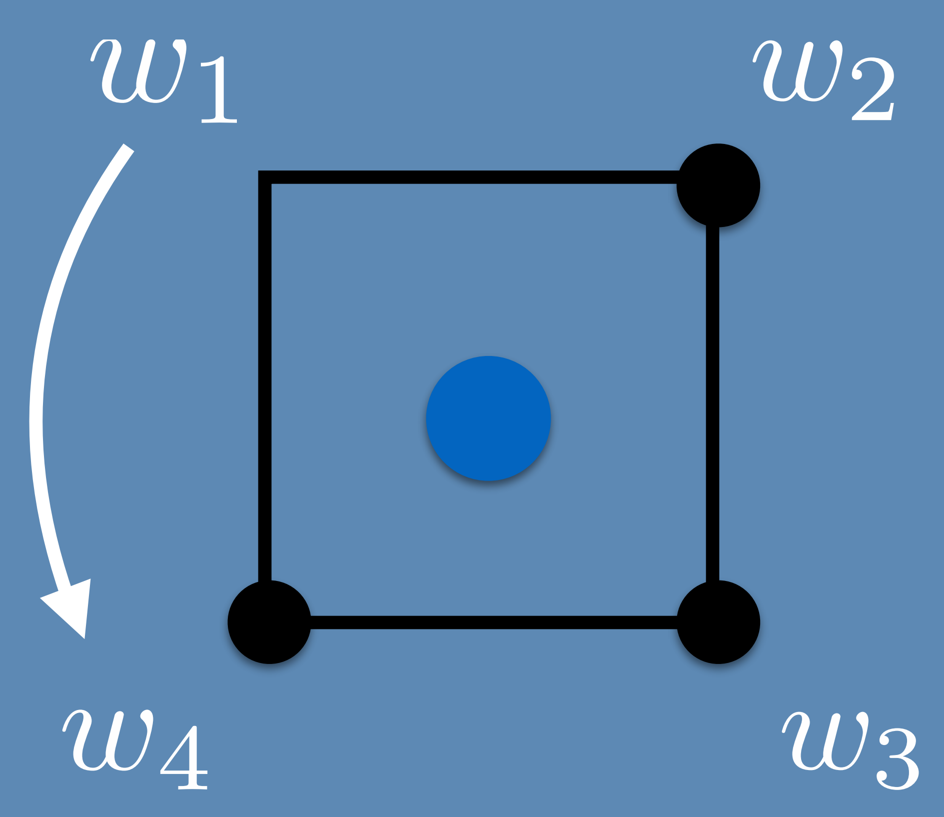

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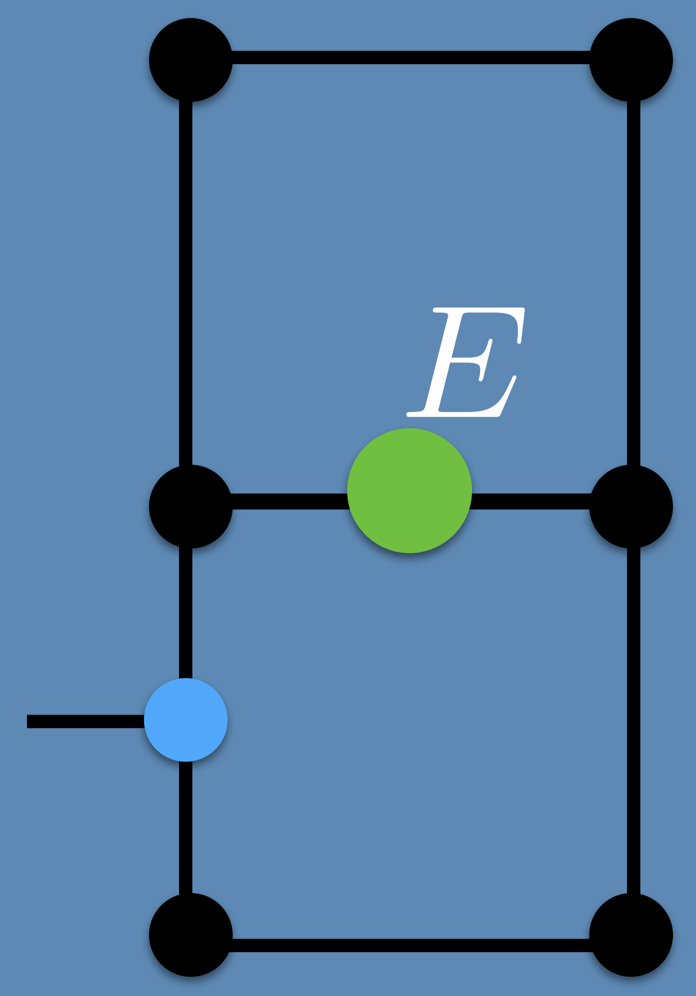
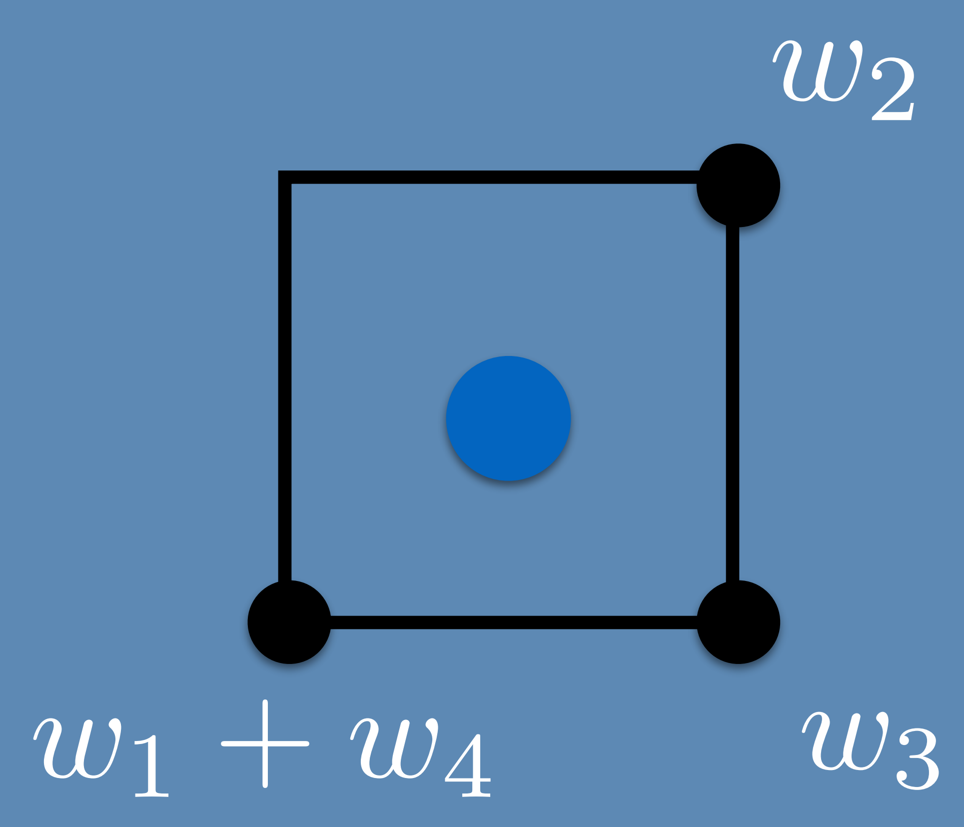
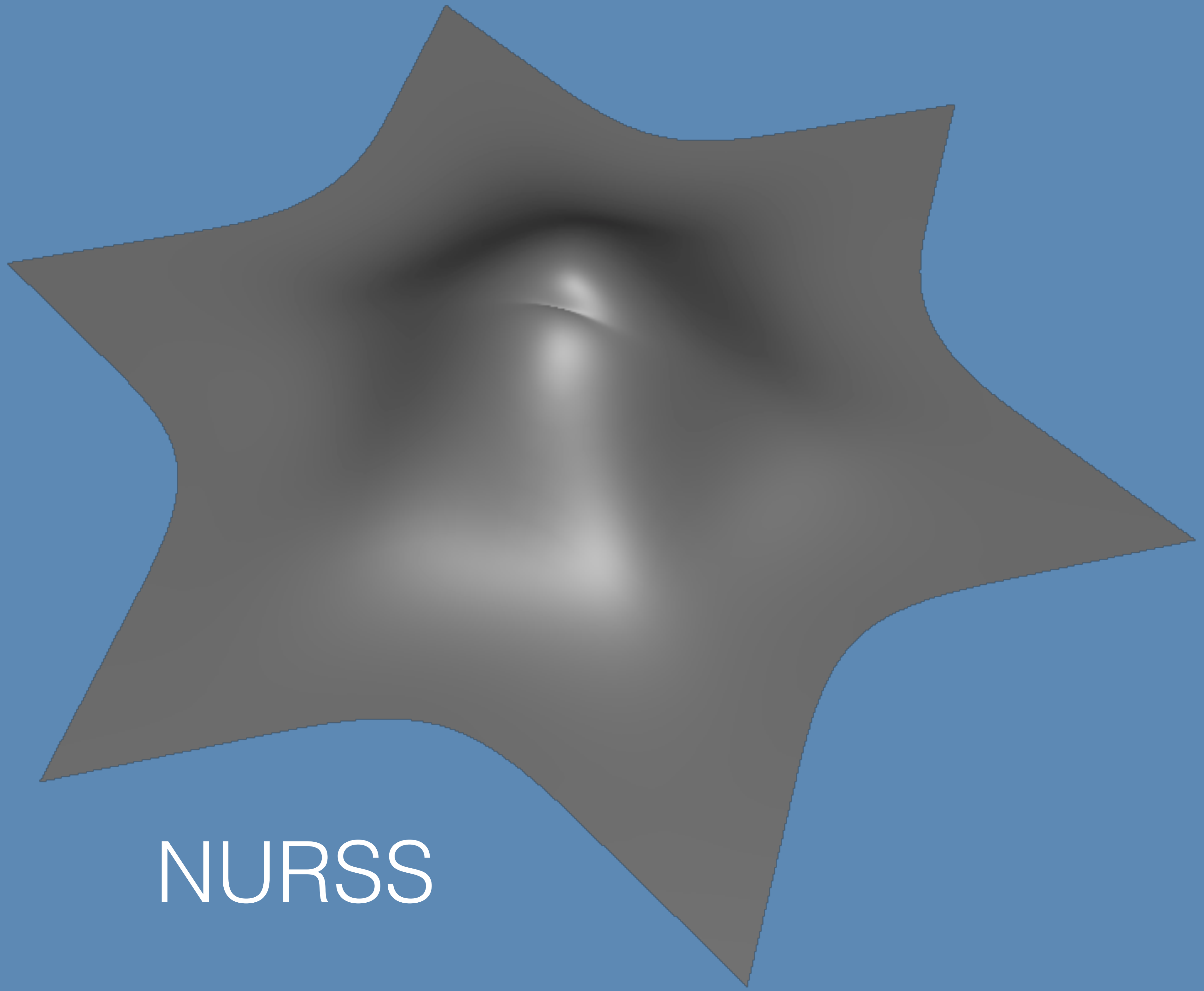


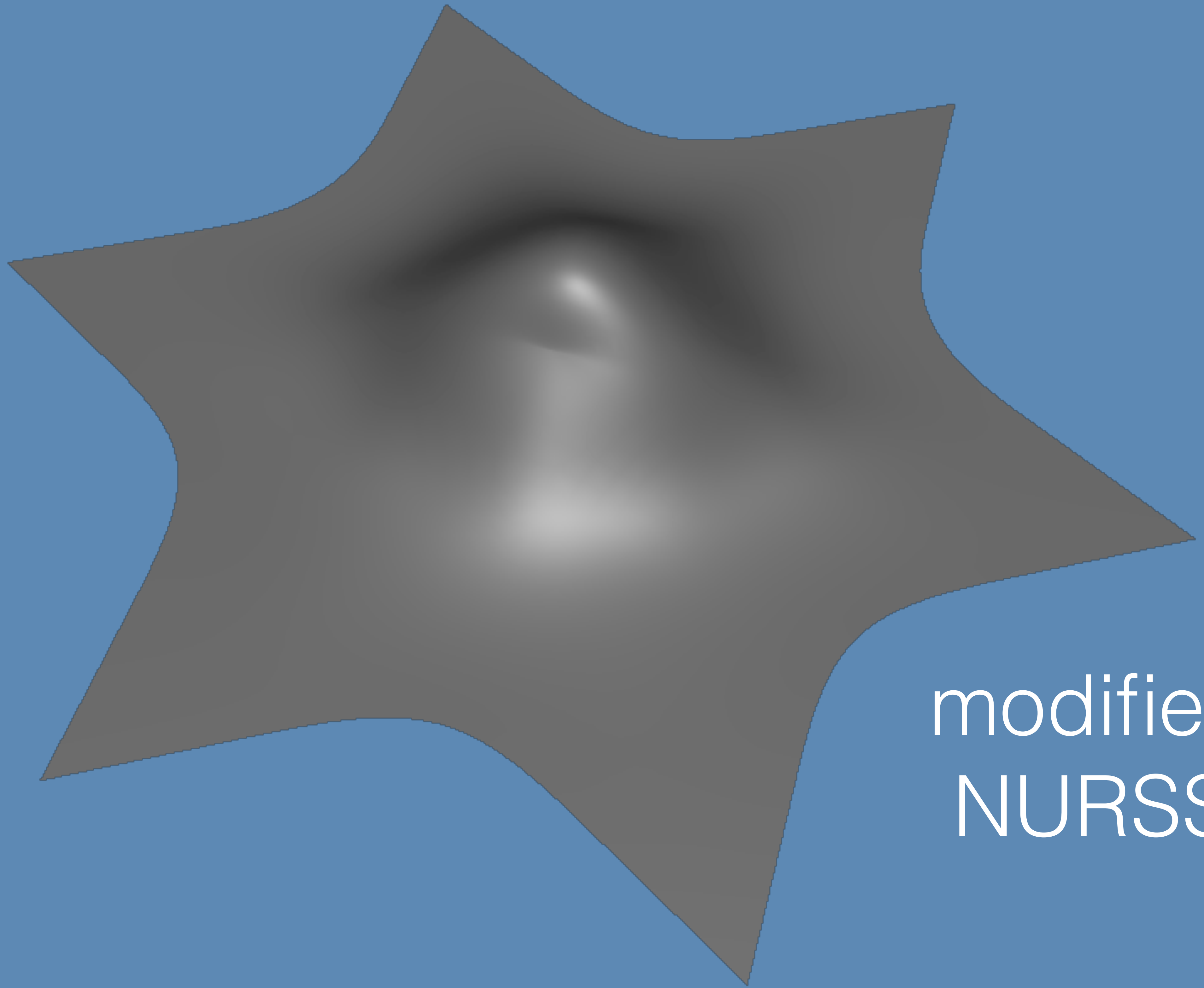



# Extraordinary Vertices

- Propose fix NURSS rules (sometimes loss of tangent plane continuity)
- Experimentally, C1 for many configurations and knot weights



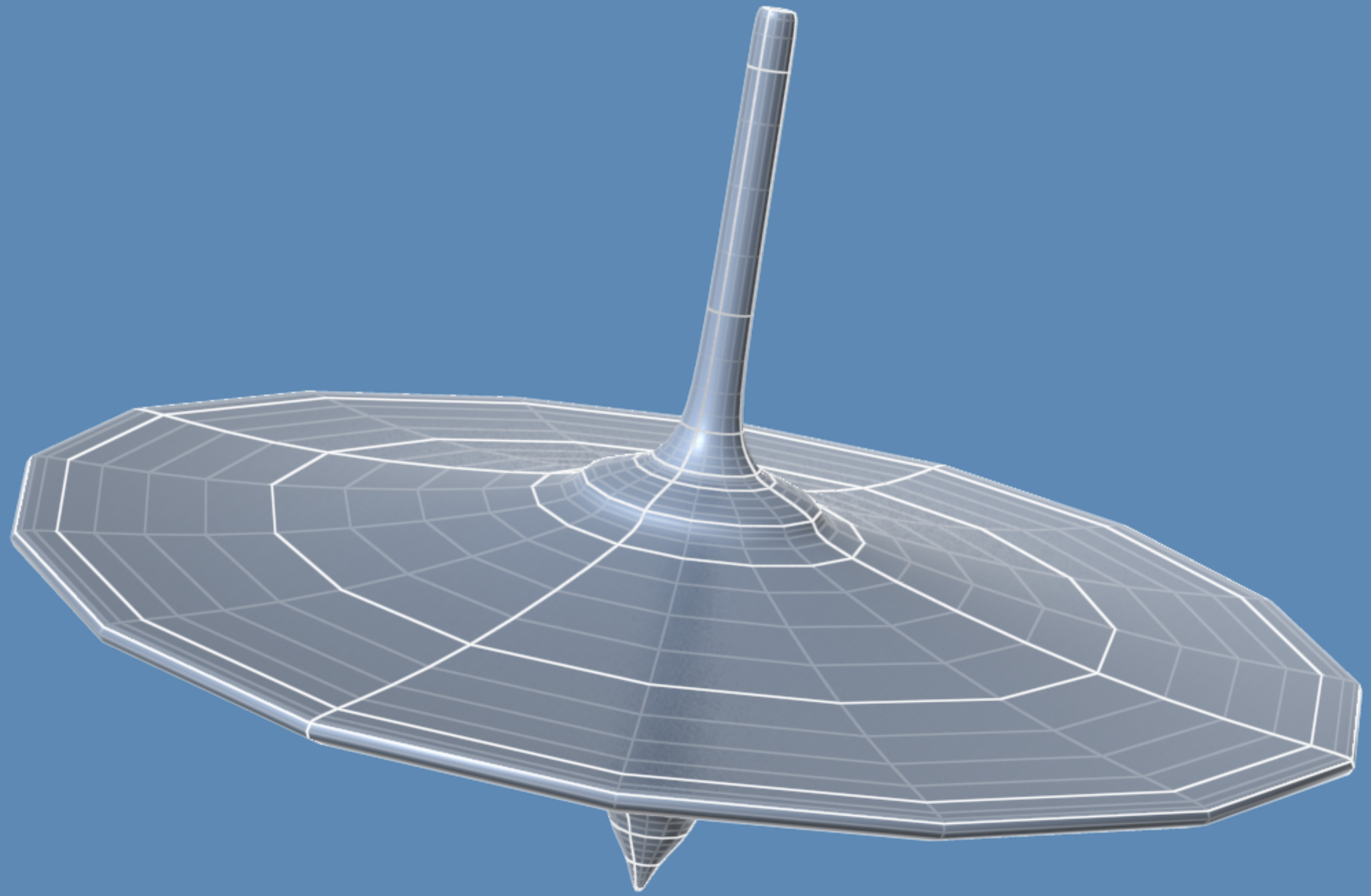
NURSS

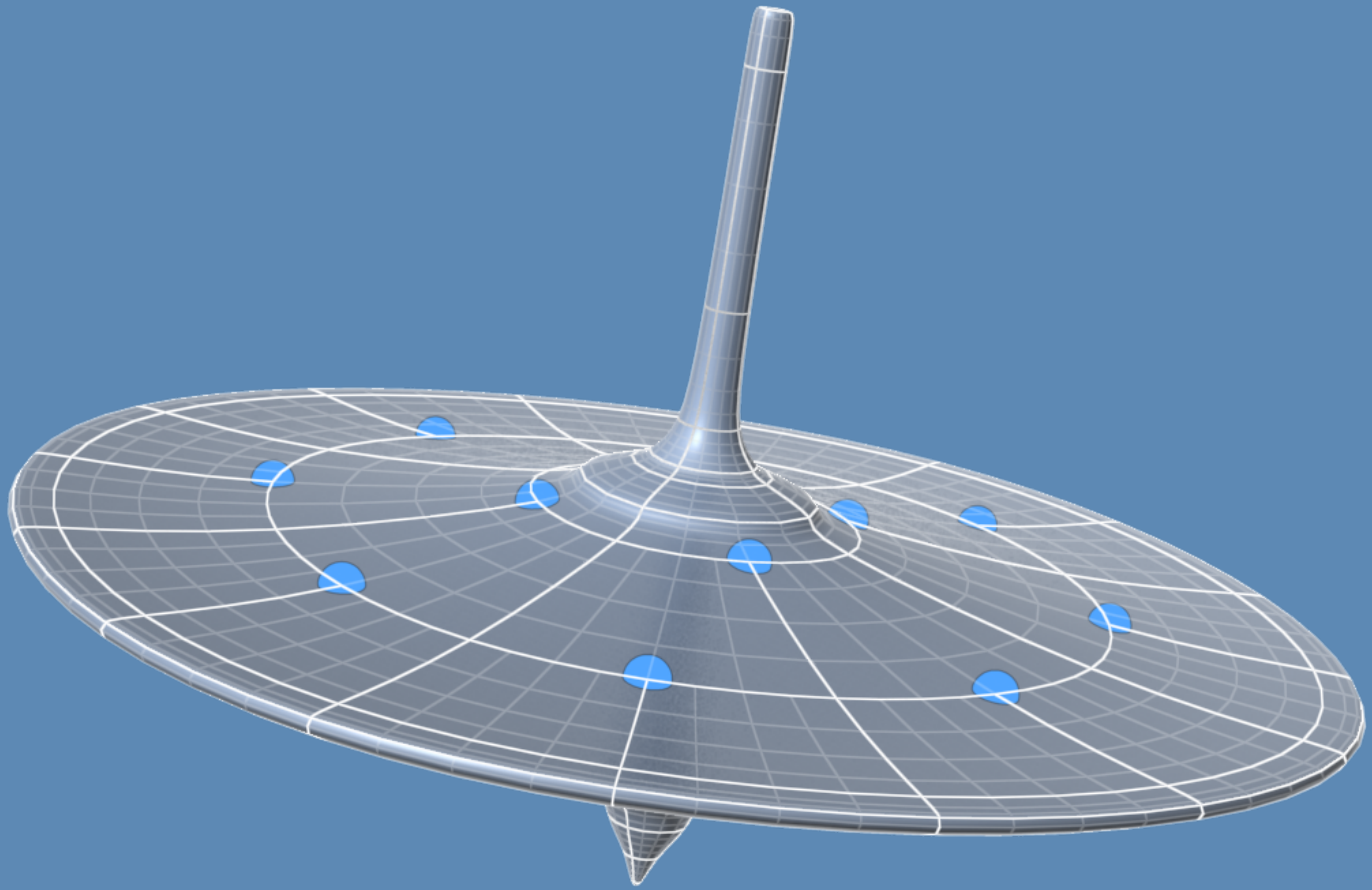


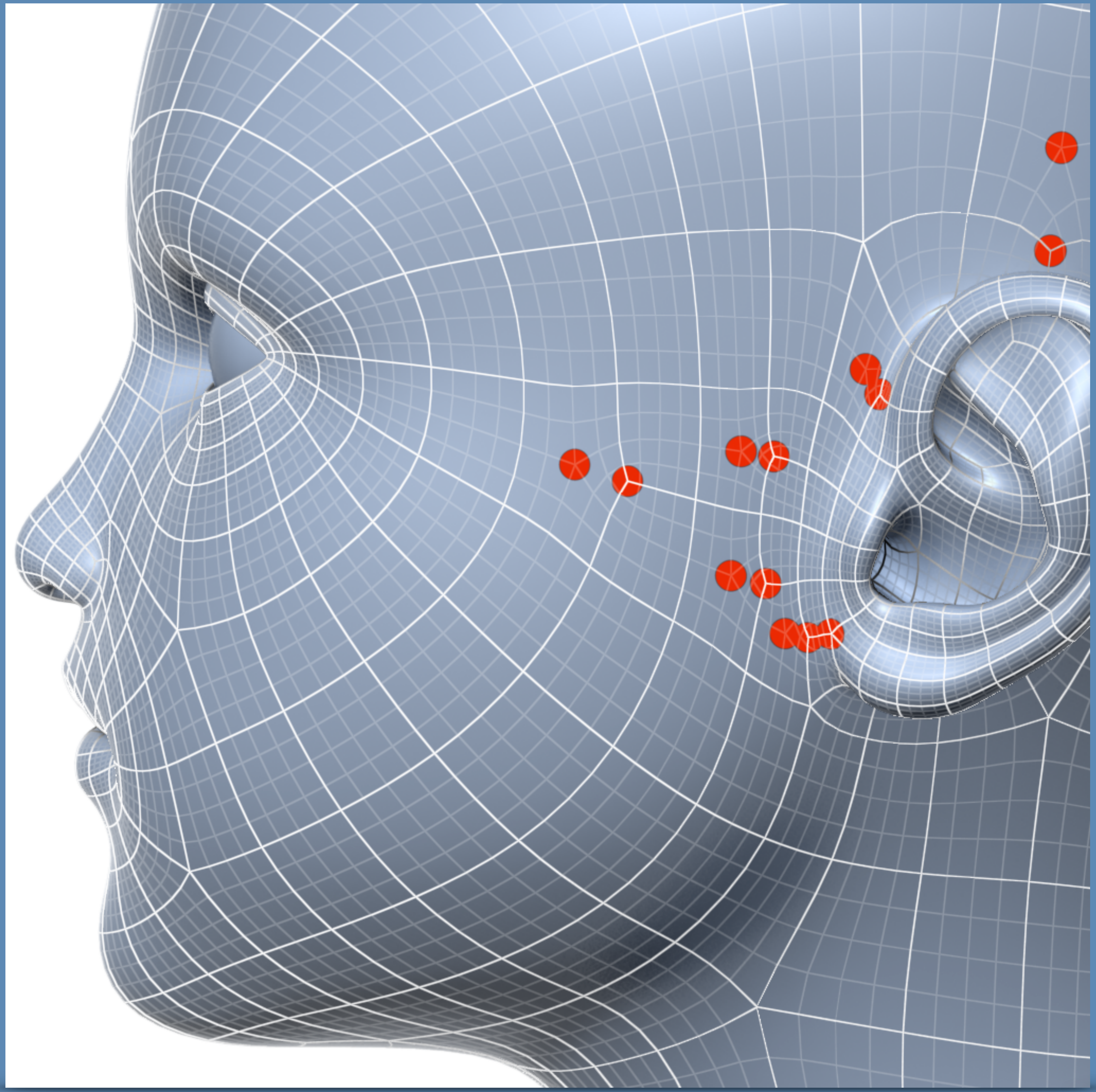
modified NURSS

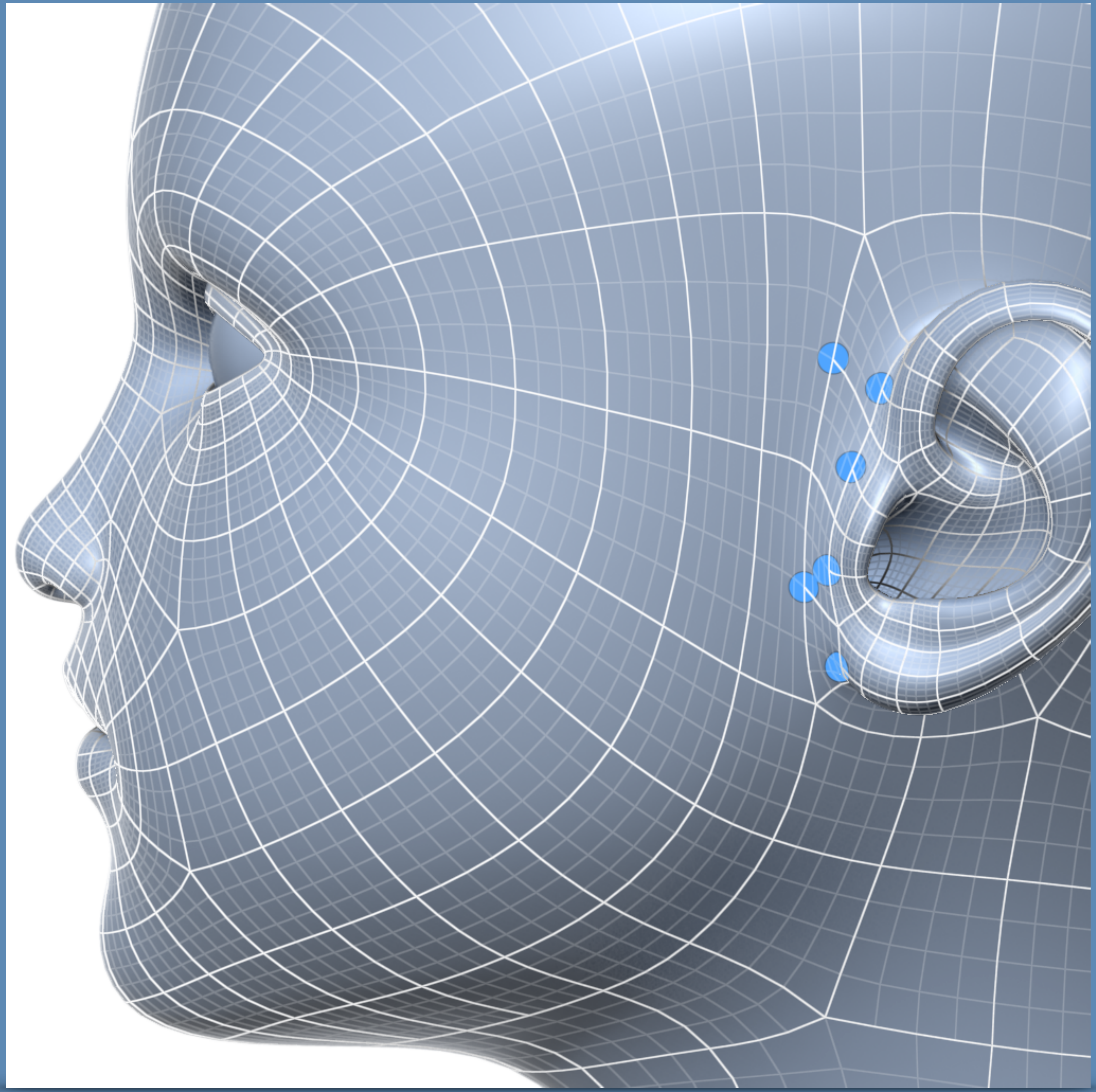
# Examples

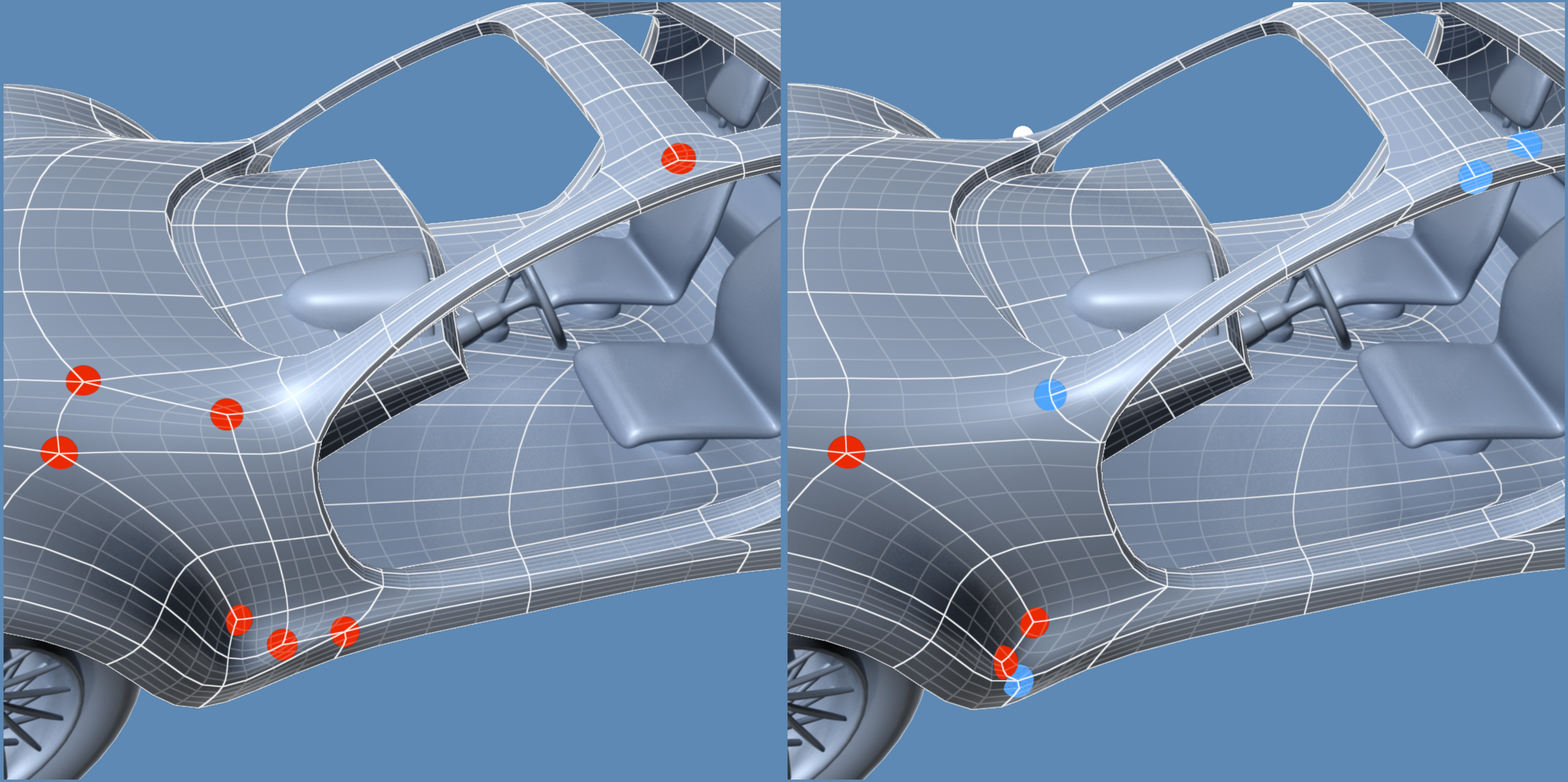
- Maya-Plugin and MATLAB scripts at
  http://denkovacs.com/publications/dyadic-t-mesh-subdivision/

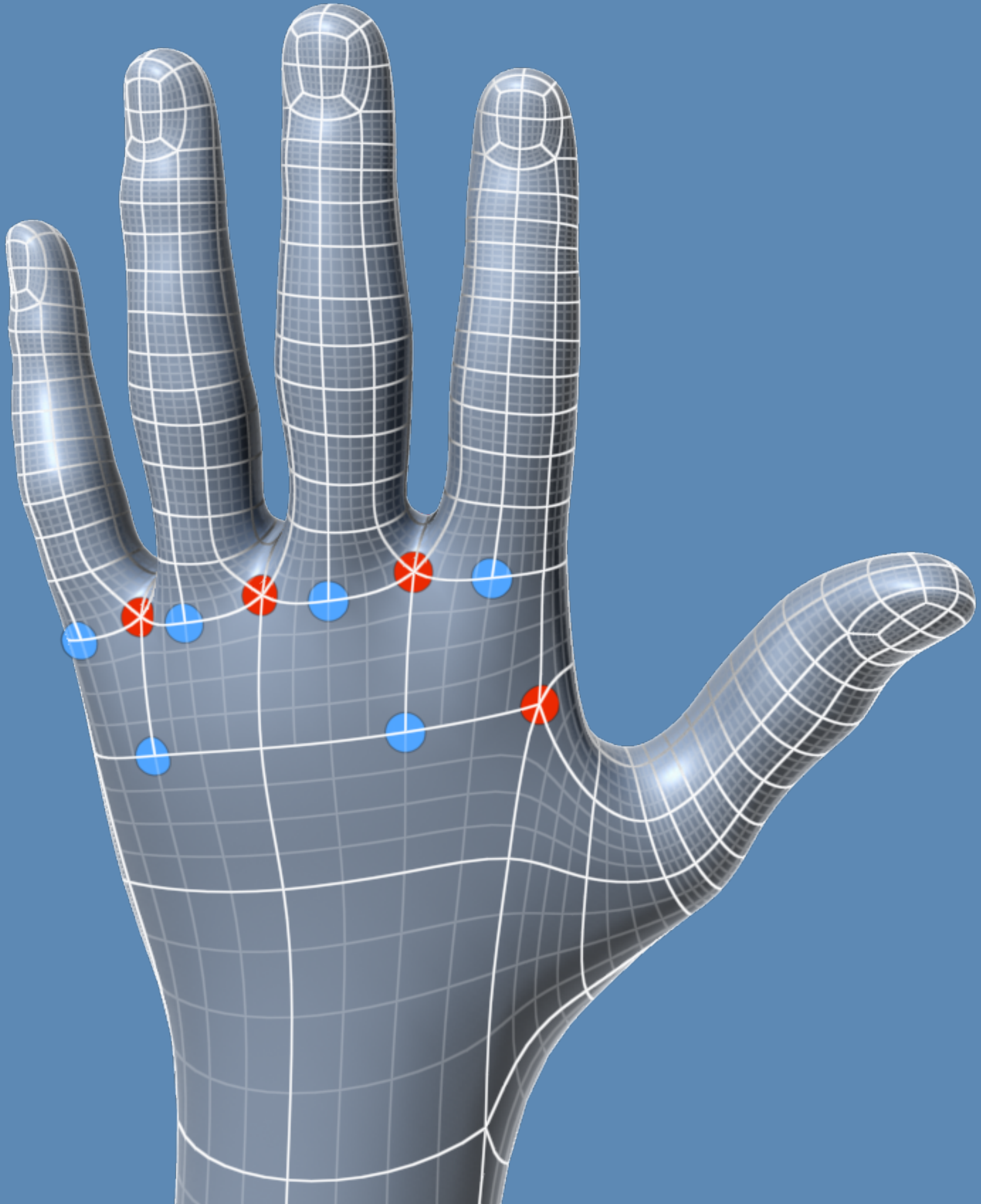












# Thanks

- Bay Raitt, Jason Mitchell, Joe Demers for great discussions
- BitmapWorld, Greg Petchovsky, Bay Raitt for models
- Jeffry Gugick for subdivision step-down guide